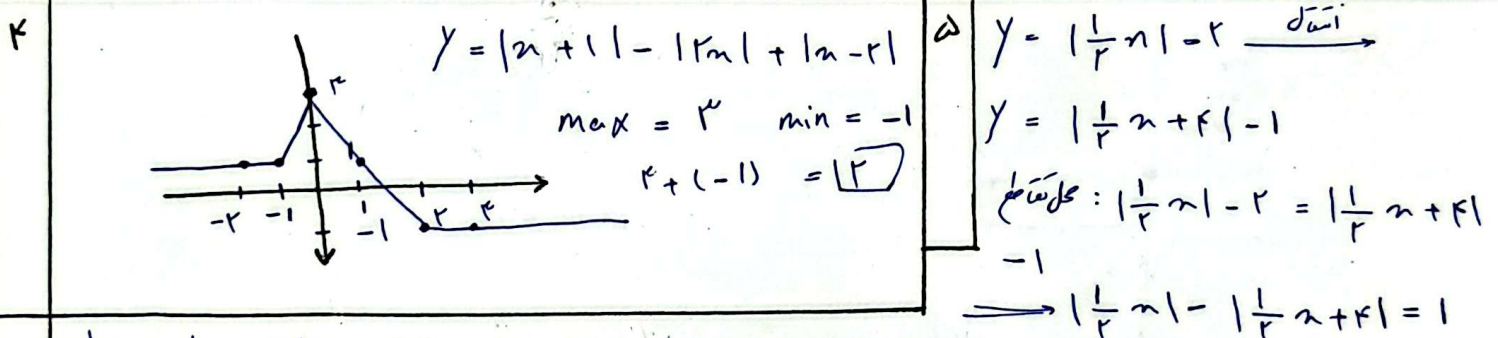


۱) تفاضل اعداد معکوسه مانند $\frac{2}{3}$ و $\frac{4}{3}$ و عدد ۱ ضرب خودشان در عدد معکوسشان ۱ می شود، اما همچنان این دو عدد معکوسه در این برابری به وجود نمی آید پس تنها:

$$\frac{1}{x-1} = |x-1| \Rightarrow (x-1)(|x-1|) = 1 \Rightarrow x-1 = 1 \Rightarrow x = 2$$

۲) الف) $-2 < \frac{x-1}{n} < 2 \Rightarrow \frac{x-1}{n} < 2 : \frac{x-1-2n}{n} < 0 \Rightarrow \frac{-1}{n} < \frac{1}{n} \Rightarrow x > 0$ II
 $I \cap II \in (\frac{1}{4}, +\infty) \Rightarrow \frac{x-1}{n} > -2 \Rightarrow \frac{x-1+2n}{n} > 0 \Rightarrow \frac{x-1}{n} > \frac{-1}{2}$ II
 $\Rightarrow \frac{x-2}{x+1} > 1 \cup \frac{x-2}{x+1} < -1 \Rightarrow \frac{x-2-2x-2}{x+1} > 0 \Rightarrow \frac{-x-4}{x+1} > 0 \Rightarrow \frac{-x}{x+1} > \frac{-4}{x+1} \Rightarrow \frac{-x}{x+1} > \frac{-4}{x+1}$
 $\Rightarrow \frac{x-2}{x+1} + 1 < \frac{x-2+2x+2}{x+1} < \frac{-1}{2} \Rightarrow \frac{3x}{x+1} < \frac{-1}{2} \Rightarrow \frac{6x}{x+1} < -1 \Rightarrow \frac{6x}{x+1} < \frac{-x-1}{x+1} \Rightarrow 6x < -x-1 \Rightarrow 7x < -1 \Rightarrow x < -\frac{1}{7}$
 $I \cup II \in (-2, \frac{1}{4}) - \{1, 2\}$

۳) $x+y > x^2 \Rightarrow x^2-x-y < 0 \Rightarrow (x-2)(x+1) < 0 \Rightarrow \frac{-2}{-1} < \frac{2}{1} \Rightarrow x \in (-2, 2)$
 $x+y < -x^2 \Rightarrow x^2+y+x < 0 \Rightarrow \emptyset$
 $|x-\alpha| < \beta \Rightarrow \alpha-\beta < x < \alpha+\beta \Rightarrow \alpha = \frac{1}{4}, \beta = \frac{5}{4} \Rightarrow \boxed{x < \frac{1}{4}}$



$\frac{1}{r}x = t \Rightarrow |t| - |t+4| = 1 \Rightarrow |\frac{1}{r}x| - |\frac{1}{r}x + 4| - 1 = 0$
 $|\frac{1}{r}x| - 1 = |\frac{1}{r}x + 4| \xrightarrow{\text{برگشت}} \frac{1}{r^2}x^2 + 1 - |x| = \frac{1}{r^2}x^2 + 19 + 4x$
 $|x| = 4x + 10 \xrightarrow{\text{برگشت}} x^2 = 16x^2 + 12 \cdot x + 100 \Rightarrow x^2 + 8x + 10 = 0$
 $\boxed{x < -3} \quad \boxed{x < -5}$

۵) الف) $D_f = \mathbb{R} \Rightarrow R_f = \mathbb{Z}$
 $\frac{1}{x} - [\frac{1}{x}] =$
 $\Rightarrow \frac{1}{x} - [1 + \frac{1}{x}] = \frac{1}{x} - [\frac{1}{x}] - 1$
 $\Rightarrow x \neq 0 \Rightarrow x \in \mathbb{Z} \Rightarrow y \in (-\frac{1}{r}, -1)$
 $\Rightarrow x \notin \mathbb{Z} \Rightarrow$

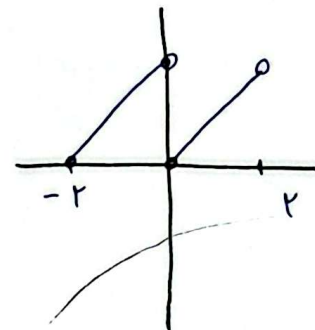
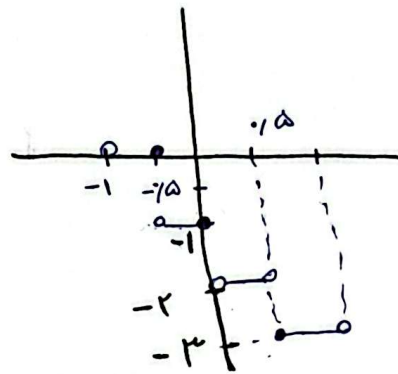
الف) $y = [-2m] - 1 \quad m \in (-1, 1)$

$m \Rightarrow -1 < m < -0.5 \Rightarrow 0$

$-0.5 < m < 0 \Rightarrow -1$

$0 < m < 0.5 \Rightarrow -2$

$0.5 < m < 1 \Rightarrow -3$



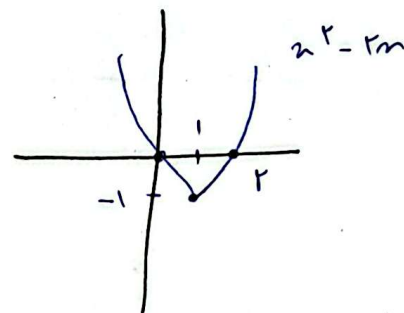
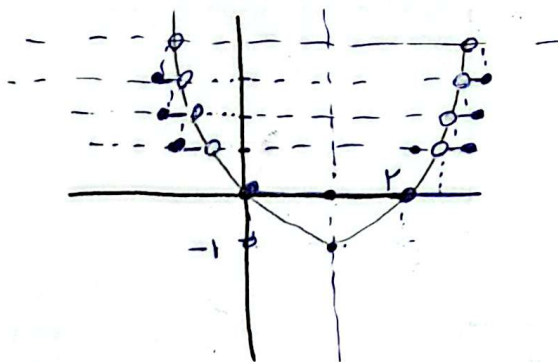
$\Rightarrow y = [-2m] - 1 \quad m \in (-1, 1)$

$a + [b] = 5, 2 \Rightarrow [b] = 5, 2 - a \quad a = k + 0, 2 \text{ (مع } 0 \leq k < 1 \text{)} \quad [b] \in [b]$

$b - [a] = 5, 2 \quad b - [k + 0, 2] = 5, 2 \Rightarrow b - k = 5, 2 \quad -k = 5, 2 - b$

$[b] = 5, 2 - k - 0, 2 = 5 - k = 5, 2 + 0, 9 \quad a = 1, 2 \quad b = 5, 2$

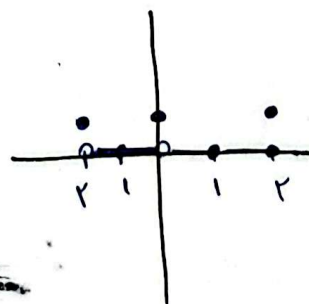
$\Rightarrow y = [m^2 - 2m] \quad m \in [-1, 2]$



الف) $m = -2 \Rightarrow y = 1 \quad -2 < m < -1 \Rightarrow y = 0$

$-1 < m < 0 \Rightarrow y = 0 \quad m = 0 \Rightarrow y = 1 \quad 0 < m < 1 \Rightarrow y = 0$

$y = 0 \Rightarrow 1 < m < 2 \Rightarrow y = 0 \quad m = 2 \Rightarrow y = 1$



-1.

$$((1+\sqrt{2})^2)^4 + ((1-\sqrt{2})^2)^4 = 191$$

$$(1+2+2\sqrt{2})^4 + (1-2-2\sqrt{2})^4 = 191$$

$$(3+2\sqrt{2}+3-2\sqrt{2})^4 - \left[3 \times 4 \times \cancel{(3-2\sqrt{2})}^1 (3+2\sqrt{2}) \right]$$

$$9^4 - 12 = 191$$

$$(1+\sqrt{2})^4 = (3+2\sqrt{2})^4 = 81 + 192\sqrt{2} + 9\cancel{(1)}^{12} + 8\sqrt{2}$$
$$= 99 + 192\sqrt{2} + 8\sqrt{2}$$

$$[99 + 192\sqrt{2} + 8\sqrt{2}] = 99 + [2(100\sqrt{2} + 4\sqrt{2})]$$

$$= 191$$