

$$\frac{1}{x-1} = |x-1|$$

$$x > 1 \rightarrow \frac{1}{x-1} = x-1 \rightarrow 1 = x^2 - 2x + 1 \rightarrow x(x-2) = 0 \rightarrow \begin{cases} x=2 \\ x=0 \text{ غلط} \end{cases}$$

$$x < 1 \rightarrow \frac{1}{x-1} = -x+1 \rightarrow 1 = -x^2 + 2x - 1 \rightarrow \Delta < 0 \quad X$$

ریشه دارد

الف) $\left| \frac{x-1}{x} \right| < 2$

$$\textcircled{1} \frac{x-1}{x} < 2 \rightarrow \frac{x-1-2x}{x} < 0 \rightarrow -\frac{1}{x} < 0 \rightarrow x > 0$$

$$\textcircled{2} \frac{x-1}{x} > -2 \rightarrow \frac{x-1+2x}{x} > 0 \rightarrow \frac{3x-1}{x} > 0 \rightarrow \frac{+}{+} \text{ یا } \frac{-}{-}$$

$$\textcircled{1} \cup \textcircled{2} \quad (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$$

$$-x \frac{x-1}{x} < 2 \rightarrow \begin{cases} \frac{1}{x} > -2 \rightarrow x > -\frac{1}{2} \\ \frac{1}{x} < 0 \rightarrow x < 0 \end{cases} \rightarrow x > -\frac{1}{2}$$

ب) $\left| \frac{x-2}{x+1} \right| > 1$

$$\textcircled{1} \frac{x-2}{x+1} > 1 \rightarrow \frac{x-2-x-1}{x+1} > 0 \rightarrow \frac{-3}{x+1} > 0 \rightarrow x < -1$$

$$\textcircled{2} \frac{x-2}{x+1} < -1 \rightarrow \frac{x-2+x+1}{x+1} < 0 \rightarrow \frac{2x-1}{x+1} < 0 \rightarrow \frac{+}{+} \text{ یا } \frac{-}{-}$$

$$\textcircled{1} \cup \textcircled{2} \quad (-1, \frac{1}{2})$$

$$|x-2| > |x+1| \rightarrow x^2 - 4x + 4 > x^2 + 2x + 1 \rightarrow -6x > -3 \rightarrow x < \frac{1}{2}$$

$$x^2 < |x+4|$$

$$\textcircled{1} x^2 < x+4 \rightarrow x^2 - x - 4 < 0 \rightarrow (x-3)(x+2) < 0 \rightarrow (-2, 3) \Rightarrow \left. \begin{aligned} \beta_2 = \frac{3}{2}, \alpha = \frac{1}{2} \\ \frac{3+(-2)}{2} = \frac{1}{2} \\ \frac{3-2}{2} = \frac{1}{2} \end{aligned} \right\} \rightarrow |x - \frac{1}{2}| < \frac{5}{2}$$

$$\textcircled{2} -x^2 > x+4 \rightarrow x^2 + x + 4 < 0 \rightarrow \Delta < 0 \quad X$$

$$y = |x+1| - |2x| + |x-2|$$

$$x \geq 2 \rightarrow x+1-2x+x-2 \rightarrow y = -1$$

$$\left. \begin{aligned} 0 \leq x < 2 \rightarrow x+1-2x-x+2 \rightarrow y = -2x+3 \quad [0, 2) \rightarrow (-1, 3] \\ -1 \leq x < 0 \rightarrow x+1+2x-x+2 \rightarrow y = 2x+3 \quad [-1, 0) \rightarrow [1, 3] \\ x \leq -1 \rightarrow -x-1+2x-x+2 \rightarrow y = 1 \end{aligned} \right\} \begin{aligned} \text{مجموعه} &= \mathbb{R} \\ \text{میزبان} &= -1 \end{aligned} \quad \left. \begin{aligned} & \\ & \end{aligned} \right\} 3-1=2$$

$$y = \left| \frac{1}{2}x \right| - \frac{1}{2} \xrightarrow{\textcircled{1}} y = \left| \frac{1}{2}x + \varepsilon \right| - \frac{1}{2} \xrightarrow{\textcircled{2}} y = \left| \frac{1}{2}x + \varepsilon \right| - \frac{1}{2} + 1 \rightarrow y' = \left| \frac{1}{2}x + \varepsilon \right| - 1$$

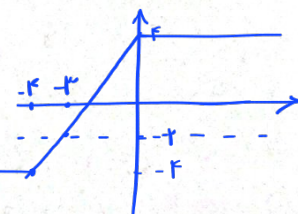
$$y' = y \rightarrow \left| \frac{1}{2}x + \varepsilon \right| - 1 = \left| \frac{1}{2}x \right| - \frac{1}{2} \Rightarrow \left| \frac{1}{2}x + \varepsilon \right| + \frac{1}{2} = \left| \frac{1}{2}x \right|$$

$$\frac{1}{2}x + \varepsilon + \frac{1}{2} = \frac{1}{2}x \rightarrow \varepsilon = 0 \quad X$$

$$\frac{1}{2}x + \varepsilon + 1 = -\frac{1}{2}x \rightarrow x + 2\varepsilon + 2 = 0 \rightarrow x = -2$$

$$y' = \left| \frac{1}{2}x + \varepsilon \right| - 1 = y$$

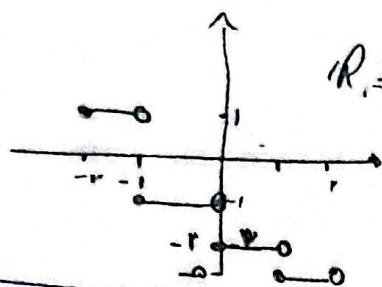
$$|x + \varepsilon| - |x| = -2$$



$$x = -2$$

$$w_1) f(x) = [|1-x| + |x+\varepsilon|]$$

$$f(x) = \frac{1}{x} - \left\lfloor \frac{x+1}{x} \right\rfloor = \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor - 1$$



$$R = \{x/x \in \mathbb{Z}, -r < x < r\}$$

$$q = \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor - 1$$

$$R = \left\{ \begin{array}{l} -r, -\varepsilon, -r \\ -r, -1, r, 1, \\ r, r, \varepsilon, r \end{array} \right\}$$

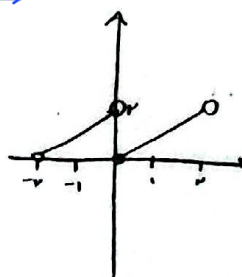
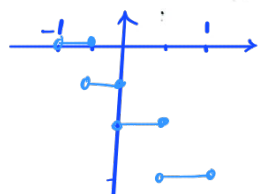
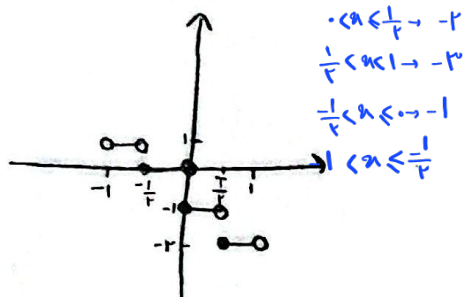
$$1/R = [-1, 0)$$

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$$w_2) q = \left\lfloor -rx \right\rfloor - 1 \quad x \in (-1, 1)$$

$$q = x - r \left\lfloor \frac{x}{r} \right\rfloor \quad x \in (-r, r)$$



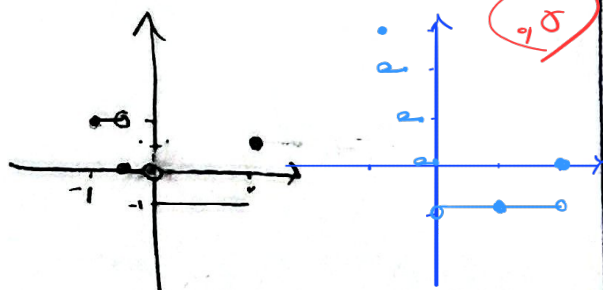
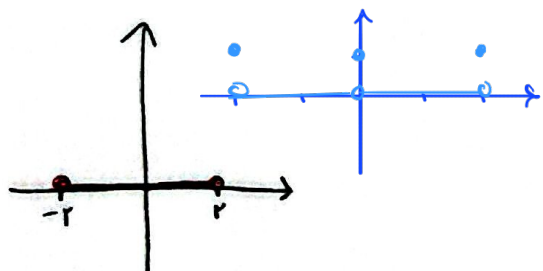
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$$q_d = [|x| - 1]$$

$$x \in [-r, r]$$

$$q = \left\lfloor x^r - rx \right\rfloor \quad x \in [-1, r]$$



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$$a + [b] = \varepsilon, r \rightarrow a = -r \rightarrow a + [b] = \varepsilon, r \rightarrow a = 1, r$$

$$b - [a] = r, \varepsilon \rightarrow b = -1, \varepsilon$$

$$b = r, \varepsilon$$

$$[a] + [b] = \varepsilon$$

$$[b] - [a] = r$$

$$a + b = 1, r + r, \varepsilon = \varepsilon, r$$

$$r[b] = r \rightarrow [b] = r$$

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9

$$(1+\sqrt{2})^4 = 19, (1-\sqrt{2})^4 = 19, \dots \quad \textcircled{1}$$

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10

$$\textcircled{1} \rightarrow 0 < 1-\sqrt{2} < 1 \xrightarrow{\text{multiplication}} 0 < (1-\sqrt{2})^4 < 1$$

$$\textcircled{2}, [(1+\sqrt{2})^4] = 19$$