

$$\frac{1}{x-1} = |x-1|$$

$$x > 1 \rightarrow \frac{1}{x-1} = x-1 \rightarrow 1 = x^2 - 2x + 1 \rightarrow x(x-2) = 0 \rightarrow \begin{cases} x=2 \\ x=0 \text{ غلط} \end{cases}$$

$$x < 1 \rightarrow \frac{1}{x-1} = -x+1 \rightarrow 1 = -x^2 + 2x - 1 \rightarrow \Delta < 0 \quad X$$

ریشه دارد

الف) $\left| \frac{x-1}{x} \right| < 2$

$$\textcircled{1} \frac{x-1}{x} < 2 \rightarrow \frac{x-1-2x}{x} < 0 \rightarrow \frac{-x-1}{x} < 0 \rightarrow x > 0$$

$$\textcircled{2} \frac{x-1}{x} > -2 \rightarrow \frac{x-1+2x}{x} > 0 \rightarrow \frac{3x-1}{x} > 0 \rightarrow x > \frac{1}{3}$$

$$\textcircled{1} \cup \textcircled{2} \quad (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$$

ب) $\left| \frac{x-2}{x+1} \right| > 1$

$$\textcircled{1} \frac{x-2}{x+1} > 1 \rightarrow \frac{x-2-x-1}{x+1} > 0 \rightarrow \frac{-2x-3}{x+1} > 0 \rightarrow \begin{matrix} (-\frac{1}{2}, -3) \\ -\frac{1}{2} \quad -3 \end{matrix}$$

$$\textcircled{2} \frac{x-2}{x+1} < -1 \rightarrow \frac{x-2+x+1}{x+1} < 0 \rightarrow \frac{2x-1}{x+1} < 0 \rightarrow \begin{matrix} \frac{1}{2} \quad -1 \\ -1 \quad \frac{1}{2} \end{matrix} \rightarrow \begin{matrix} (-\frac{1}{2}, \frac{1}{2}) \\ (-\frac{1}{2}, \frac{1}{2}) \end{matrix}$$

$$\textcircled{1} \cup \textcircled{2} \quad (-\frac{1}{2}, \frac{1}{2})$$

$$x^2 < |x+4|$$

$$\textcircled{1} x^2 < x+4 \rightarrow x^2 - x - 4 < 0 \rightarrow (x-3)(x+2) < 0 \rightarrow (-2, 3) \Rightarrow \left. \begin{matrix} \beta = \frac{3}{2}, \alpha = \frac{1}{2} \\ \frac{3+(-2)}{2} = \frac{1}{2} \\ \frac{3-2}{2} = \frac{1}{2} \end{matrix} \right\} \rightarrow |x - \frac{1}{2}| < \frac{1}{2}$$

$$\textcircled{2} -x^2 > x+4 \rightarrow x^2 + x + 4 < 0 \rightarrow \Delta < 0 \quad X$$

$$y = |x+1| - |x| + |x-2|$$

$$x \geq 2 \rightarrow x+1 - x + x-2 \rightarrow y = x-1$$

$$\left. \begin{matrix} 0 \leq x < 2 \rightarrow x+1 - x - x+2 \rightarrow y = -x+3 \quad [0, 2] \rightarrow (-1, 3] \\ -1 \leq x < 0 \rightarrow x+1 + x - x+2 \rightarrow y = x+2 \quad [-1, 0] \rightarrow [1, 2] \\ x < -1 \rightarrow -x-1 + x - x+2 \rightarrow y = -x+1 \end{matrix} \right\} \begin{matrix} \text{مجموعه} = \mathbb{R} \\ \text{میزان} = -1 \end{matrix} \quad \left. \begin{matrix} 3-1=2 \\ -1-1=-2 \end{matrix} \right\}$$

$$y = \left| \frac{1}{2}x \right| - \frac{1}{2} \xrightarrow{\textcircled{1}} y = \left| \frac{1}{2}x + \varepsilon \right| - \frac{1}{2} \xrightarrow{\textcircled{2}} y = \left| \frac{1}{2}x + \varepsilon \right| - \frac{1}{2} + 1 \rightarrow y' = \left| \frac{1}{2}x + \varepsilon \right| - 1$$

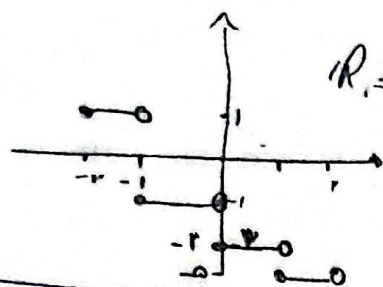
$$y' = y \rightarrow \left| \frac{1}{2}x + \varepsilon \right| - 1 = \left| \frac{1}{2}x \right| - \frac{1}{2} \Rightarrow \left| \frac{1}{2}x + \varepsilon \right| + \frac{1}{2} = \left| \frac{1}{2}x \right|$$

$$\frac{1}{2}x + \varepsilon + \frac{1}{2} = \frac{1}{2}x \rightarrow \varepsilon = 0 \quad X$$

$$\frac{1}{2}x + \varepsilon + 1 = -\frac{1}{2}x \rightarrow x + 2\varepsilon + 2 = 0 \rightarrow x = -2 - 2\varepsilon$$

$$iii) f(x) = [1-x] + [x+\varepsilon]$$

$$f(x) = \frac{1}{x} - \left[\frac{x+1}{x} \right] = \frac{1}{x} - \left[\frac{1}{x} \right] - 1$$



$$R_1 = \{x/x \in \mathbb{Z}, -1 \leq x \leq 1\}$$

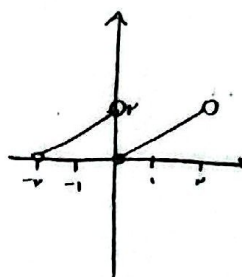
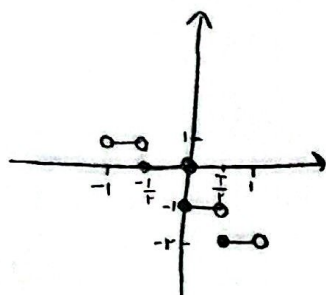
$$y_2 = \frac{1}{x} - \left[\frac{1}{x} \right] - 1$$

$$R_2 = \left\{ \begin{array}{l} -1, -\varepsilon, -r \\ -r, -1, \varepsilon, 1 \\ r, r, \varepsilon, 0 \end{array} \right\}$$

$$IR = [-1, 0)$$

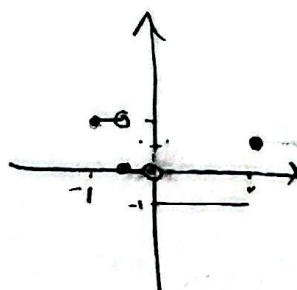
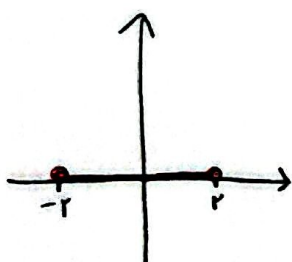
$$iv) y_1 = [-rx] - 1 \quad x \in (-1, 1)$$

$$y_2 = x - r \left[\frac{x}{r} \right] \quad x \in (-r, r)$$



$$v) y_1 = [1/x - 1] \quad x \in [r, r]$$

$$y_2 = [x^r - rx] \quad x \in [1, r]$$



$$a + [b] = \varepsilon, r \rightarrow a = -r \rightarrow a + [b] = \varepsilon, r \rightarrow a = 1, r$$

$$b - [a] = r, \varepsilon \rightarrow b = -1, \varepsilon$$

$$b = r, \varepsilon$$

$$[a] + [b] = \varepsilon$$

$$[b] - [a] = r$$

$$a + b = 1, r + r, \varepsilon = \varepsilon, r$$

$$r[b] = r \rightarrow [b] = r$$

$$(1+\sqrt{2})^n = 19n - (1-\sqrt{2})^n \xrightarrow{①} (1+\sqrt{2})^n = 19n, \dots \quad ②$$

$$① \rightarrow 0 < 1-\sqrt{2} < 1 \xrightarrow{②} 0 < (1-\sqrt{2})^n < 1$$

$$②, [(1+\sqrt{2})^n] = 19n$$