

Date:

Sub:

المعادلات التفاضلية

19, 75

$$\frac{1}{x-1} = |x-1| \xrightarrow{x-1=t} \frac{1}{t} = |t| \quad x \neq 1 \quad (1)$$

$$\Rightarrow \text{if } t > 0 \Rightarrow t = |t| \Rightarrow \frac{1}{t} = t \Rightarrow t^2 = 1 \Rightarrow t = 1$$

$$\Rightarrow \text{if } t < 0 \Rightarrow |t| = -t \Rightarrow \frac{1}{t} = -t \Rightarrow t^2 = -1 \rightarrow \text{مخفوق}$$

$$\Rightarrow \text{مفاد } t = 1 \Rightarrow x - 1 = 1 \Rightarrow x = 2$$

(2)

$$\text{الف) } \left| \frac{2x-1}{x} \right| < 2 \quad x \neq 0 \Rightarrow -2 < \frac{2x-1}{x} < 2$$

$$\Rightarrow \left\{ \begin{array}{l} \frac{2x-1}{x} < 2 \Rightarrow \frac{2x-1-2x}{x} < 0 \Rightarrow \frac{-1}{x} < 0 \Rightarrow x > 0 \\ \frac{2x-1}{x} > -2 \Rightarrow \frac{2x-1+2x}{x} > 0 \Rightarrow \frac{4x-1}{x} > 0 \Rightarrow 4x-1 > 0 \end{array} \right.$$

$$, x > 0 \Rightarrow x > \frac{1}{4} \quad \text{و} \quad 4x-1 < 0 \Rightarrow x < \frac{1}{4}$$

$$\Rightarrow x > \frac{1}{4} \Rightarrow \left(\frac{1}{4}, +\infty \right)$$

(3)

$$\text{ب) } \left| \frac{x-2}{2x+1} \right| > 1 \quad 2x+1 \neq 0 \Rightarrow 2x \neq -1 \Rightarrow x \neq -\frac{1}{2}$$

$$\Rightarrow \left\{ \begin{array}{l} \frac{x-2}{2x+1} > 1 \Rightarrow \frac{x-2-(2x+1)}{2x+1} > 0 \Rightarrow \frac{-x-3}{2x+1} > 0 \Rightarrow (-3, -\frac{1}{2}) \\ \frac{x-2}{2x+1} < -1 \Rightarrow \frac{x-2+(2x+1)}{2x+1} < 0 \Rightarrow \frac{3x-1}{2x+1} < 0 \Rightarrow (-\frac{1}{2}, \frac{1}{3}) \end{array} \right.$$

$$\Rightarrow (-3, -\frac{1}{2}) \cup (-\frac{1}{2}, \frac{1}{3})$$

$$x^r < |x+y|$$

②

$$\Rightarrow \text{if } x+y \geq 0 \Rightarrow x \geq -y \Rightarrow x^r < x+y \Rightarrow x^r - x - y < 0$$

⑤

$$\Rightarrow (x-r)(x+r) < 0 \Rightarrow -r < x < r$$

$$\Rightarrow \text{if } x+y < 0 \Rightarrow x < -y \Rightarrow x^r < -(x+y) \Rightarrow x^r + x + y < 0$$

$$\Rightarrow \bar{0} \bar{0} \bar{2}$$

$$\Rightarrow (-r, r) \quad |x-\alpha| < \beta \Rightarrow \alpha = \frac{r+(-r)}{r} = \left[\frac{1}{r} \right]$$

$$y = |x+1| - |rx| + |x-r|$$

③

$$\text{if } x < -1 \Rightarrow y = 1$$

⑤

$$\text{if } -1 \leq x < 0 \Rightarrow y = rx + r$$

$$\text{if } 0 \leq x < r \Rightarrow y = -rx + r$$

$$\text{if } x \geq r \Rightarrow y = -1$$

$$\Rightarrow \left. \begin{array}{l} y_{\max} = r \quad (x=0) \\ y_{\min} = -1 \quad (x \geq r) \end{array} \right\} \Rightarrow r+(-1)=r$$

$$y = \left| \frac{1}{r}x \right| - r \rightarrow \text{تابع } y = \left| \frac{1}{r}(x+r) \right| - r + 1$$

④

$$\Rightarrow y = \left| \frac{1}{r}(x+r) \right| - 1$$

$$\left| \frac{1}{r}x \right| - r = \left| \frac{1}{r}(x+r) \right| - 1 \Rightarrow \left| \frac{1}{r}x \right| = \left| \frac{1}{r}(x+r) \right| + 1$$

⑤

$$\text{if } x \geq 0 \Rightarrow \frac{1}{r}x = \frac{1}{r}(x+r) + 1 \Rightarrow \frac{1}{r}x = \frac{1}{r}x + r + 1$$

$$\rightarrow \bar{0} \bar{0} \bar{2}$$

$$\text{if } x < -r \Rightarrow -\frac{1}{r}x = -\frac{1}{r}(x+r) + 1 \Rightarrow -\frac{1}{r}x = -\frac{1}{r}x - r + 1 \rightarrow \bar{0} \bar{0} \bar{2}$$

$$\text{if } -r \leq x < 0 \Rightarrow -\frac{1}{r}x = \frac{1}{r}(x+r) + 1 \Rightarrow -x = x + r \Rightarrow x = -r \rightarrow \bar{0} \bar{0}$$

$$\text{ii) } f(x) = [1-x] - |x+\varepsilon|$$

(1, 0) (2)

$$\text{if } x < -\varepsilon \Rightarrow f(x) = (1-x) - (-(x+\varepsilon)) = 1-x+x+\varepsilon = \varepsilon$$

$$\Rightarrow f(x) = \varepsilon$$

$$\text{if } -\varepsilon \leq x < 1 \Rightarrow f(x) = (1-x) - (x+\varepsilon) = -2x - \varepsilon \Rightarrow$$

$$\bullet f(-\varepsilon) = -2(-\varepsilon) - \varepsilon = 1 - \varepsilon = \varepsilon$$

$$\text{if } x \geq 1 \Rightarrow \text{[scribbled out]} f(x) = (x-1) - (x+\varepsilon) = -\varepsilon$$

$$\Rightarrow f(x) = -\varepsilon$$

$$\Rightarrow f(x) \text{ is } = \{-\varepsilon, -\varepsilon, -\varepsilon, -\varepsilon, -1, 0, 1, \varepsilon, \varepsilon, \varepsilon\}$$

$$\rightarrow \frac{1}{x} = \left[\frac{x+1}{x} \right] \quad x \neq 0$$

$$\Rightarrow \text{[scribbled out]} y = \frac{x+1}{x} = 1 + \frac{1}{x} \Rightarrow f(x) = \frac{1}{x} - \left[1 + \frac{1}{x} \right] =$$

$$\left(\frac{1}{x} + 1 - 1 \right) - \left[1 + \frac{1}{x} \right] = \frac{1}{x} - \left[1 + \frac{1}{x} \right]$$

$$f(x) = -\left\{ 1 + \frac{1}{x} \right\} \in (-1, 0] \Rightarrow \overline{(-1, 0]} \quad [-1, 0]$$

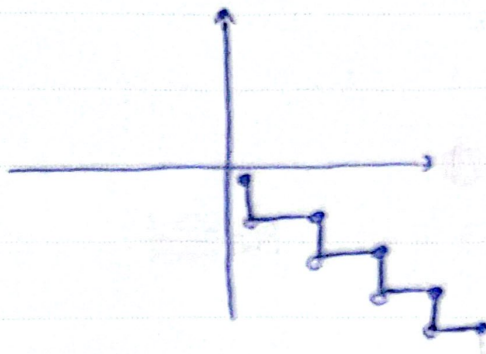
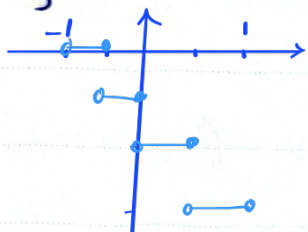
الف) $y = [-2x] - 1 \Rightarrow$

$-\infty < x \leq -\frac{1}{2} \rightarrow -1$

$-\frac{1}{2} < x \leq 0 \rightarrow -2$

$0 < x \leq \frac{1}{2} \rightarrow -1$

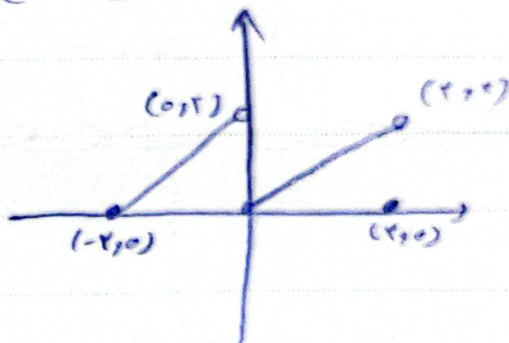
$\frac{1}{2} < x \leq \infty \rightarrow -2$



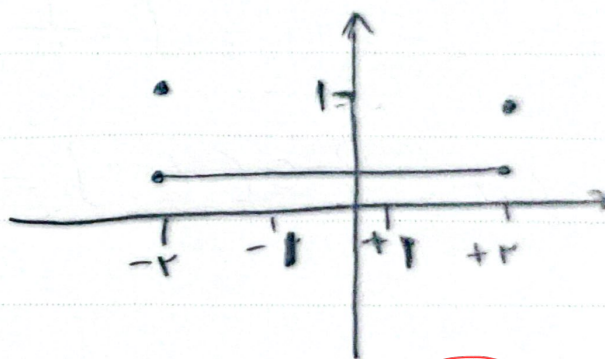
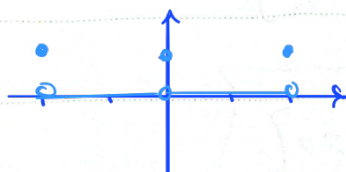
①

ب) $y = x - 2 \left[\frac{x}{2} \right] \quad x \in (-2, 2)$

①



ج) $y = [|x| - 1] \quad x \in [-2, 2]$

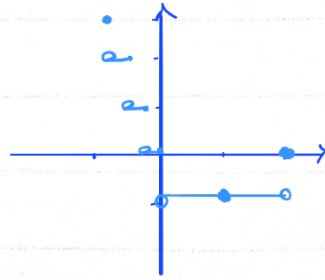
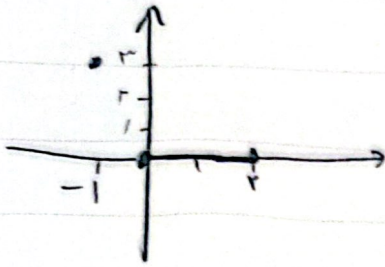


①

①

$$y = [x^T - rx]$$

$$x \in [-1, 1]$$



$$a + [b] = r, r \Rightarrow a = r, r - [b]$$

(9)

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$$b - [a] = r, r \Rightarrow b = r, r + [a]$$

$$\Rightarrow b = r, r + [r, r - [b]]$$

$$\Rightarrow a = 1, r, b = r, r \Rightarrow a + b = 1, r + r, r = r, r$$

(10)

$$(1 + \sqrt{r})^4 + (1 - \sqrt{r})^4 = 191$$

$$\Rightarrow (1 + \sqrt{r})^4 = [191 - (1 - \sqrt{r})^4] \Rightarrow \boxed{191}$$

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