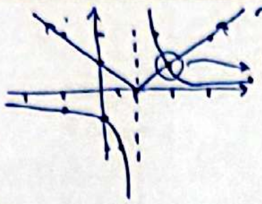
①

$\frac{1}{x-1} = |x-1| \Rightarrow$



$\frac{1}{x-1}$:
x	0	1	3	2
y	-1		$\frac{1}{2}$	1

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②

$\left|\frac{2x-1}{x}\right| < 2 \Rightarrow \begin{cases} \frac{2x-1}{x} < 2 \Rightarrow \frac{2x-1-2x}{x} < 0 \Rightarrow \frac{-1}{x} < 0 \quad \frac{\quad 0 \quad}{+ \quad | \quad -} \Rightarrow x \in (0, +\infty) \text{ ①} \\ -2 < \frac{2x-1}{x} \Rightarrow 0 < \frac{2x-1+2x}{x} \Rightarrow 0 < \frac{4x-1}{x} \Rightarrow \frac{0 \quad \frac{1}{4}}{+ \quad - \quad +} \Rightarrow x \in (-\infty, 0) \cup (\frac{1}{4}, +\infty) \text{ ②} \end{cases}$

$x \in (\frac{1}{4}, +\infty)$

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$\left|\frac{x-2}{2x+1}\right| > 1 \Rightarrow \begin{cases} \frac{x-2}{2x+1} > 1 \Rightarrow \frac{x-2-2x-1}{2x+1} > 0 \Rightarrow \frac{-3-x}{2x+1} > 0 \Rightarrow \frac{-3 \quad -\frac{1}{2}}{- \quad + \quad -} \Rightarrow x \in (-3, -\frac{1}{2}) \text{ ①} \\ \frac{x-2}{2x+1} < -1 \Rightarrow \frac{x-2+2x+1}{2x+1} < 0 \Rightarrow \frac{3x-1}{2x+1} < 0 \Rightarrow \frac{-\frac{1}{2} \quad \frac{1}{3}}{+ \quad - \quad +} \Rightarrow x \in (-\frac{1}{2}, \frac{1}{3}) \text{ ②} \end{cases}$

① $\cup$ ② $\Rightarrow x \in \cancel{(-\frac{1}{2}, -\frac{1}{2}) \Rightarrow x = \{-\frac{1}{2}\}}$

$x \in (-3, \frac{-1}{2}) \cup (\frac{-1}{2}, \frac{1}{3})$

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③

$x^2 < |x+6|$, $|x-\alpha| < \beta$

$|x-\alpha| < \beta \Rightarrow \begin{cases} x-\alpha < \beta \Rightarrow x < \beta+\alpha \\ -\beta < x-\alpha \Rightarrow x > \alpha-\beta \\ \alpha-\beta < x < \beta+\alpha \end{cases} \Rightarrow$

$x^2 < x+6 \to x^2-x-6 < 0 \Rightarrow \frac{-2 \quad 3}{+ \quad - \quad +} \Rightarrow -2 < x < 3$

$x+6 < -x^2 \Rightarrow x^2+x+6 < 0$ } غ ق ق

همواره + $\Longleftarrow \Delta < 0$, $a > 0$

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$\begin{cases} \alpha-\beta = -2 \\ \alpha+\beta = 3 \end{cases}$

$2\alpha = 1 \Rightarrow \alpha = \frac{1}{2}$, $\beta = \frac{5}{2}$

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④

$y = |x+1| - |2x| + |x-2|$

max: $|x+1| - |\underset{0}{2x}| + |x-2| \Rightarrow x=0 \Rightarrow |1| - |0| + |-2| = 3$ ✓ $\Rightarrow$ max

min: 0 ~~max~~ $0 \Rightarrow |\underbrace{x+1}_{\substack{0 \\ x=-1}}| - |2x| + |\underbrace{x-2}_{\substack{0 \\ x=2}}| \Rightarrow$
$x=2 \to |3| - |4| + |0| \Rightarrow 3-4 = -1$ ✓ min
$x=-1 \to |0| - |-2| + |-3| \Rightarrow -2+3 = 1$

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min + max $= -1 + 3 = \boxed{2}$

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⑤

$y = |\frac{1}{4}x| - 2 \xrightarrow[\text{منفی}]{\text{۸ واحد به طرف}} y = |\frac{1}{2}x + 4| - 2 \xrightarrow[\text{y مثبت}]{\text{۱ واحد طرف}} y = |\frac{1}{2}x + 4| - 1$

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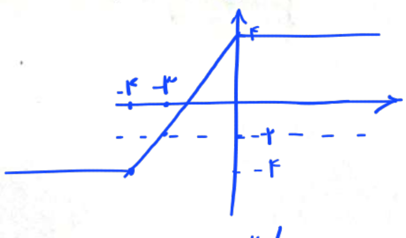
$|\frac{1}{2}x| - 2 = |\frac{1}{4}x + 4| - 1 \Rightarrow |\underbrace{\frac{1}{4}x}_{0}| - |\underbrace{\frac{1}{2}x+4}_{-8}| = 1 \Rightarrow$

	-8		0	
$-\frac{1}{2}x + \frac{1}{2}x + 4$		$-\frac{1}{2}x - \frac{1}{2}x - 4$		$\frac{1}{2}x - \frac{1}{2}x - 4$
4		$-x-4$		$\boxed{-4}$

$-x-4 = 1 \Rightarrow -x = 5 \Rightarrow x = -5$

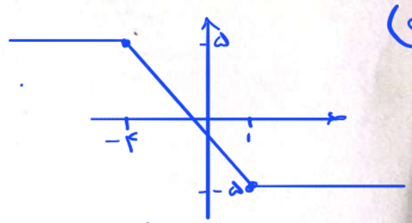
$y' = |\frac{1}{2}x+2| - 1 = y$

$|x+4| - |x| = -2$



$x = -3$

۶ الف)



$R_f = \{-5, -4, \dots, 4, 5\}$

۶ الف) $f(x) = [\,|1-x| - |x+4|\,] \Rightarrow$

-4		1
$-1+x+x+4$	$-1+x-x-4$	$1-x-x-4$
$2x+3$	-5	$-2x-3$

$\Rightarrow R_f = \mathbb{Z} - (-5, +\infty)$

ب) $f(x) = \frac{1}{x} - \left[\frac{x+1}{x}\right] \Rightarrow \begin{cases} \frac{1}{x} - 1 & x > 0 \Rightarrow R_f = [0, +\infty) \; ① \\ \frac{1}{x} + 1 & x < 0 \Rightarrow R_f = (-\infty, 0] \; ② \end{cases}$ $① \cup ② \Rightarrow R_f = \mathbb{R}$

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ب) $f(x) = \frac{1}{x} - \left[\frac{1}{x}\right] - 1 \rightarrow R_f = [-1, 0)$

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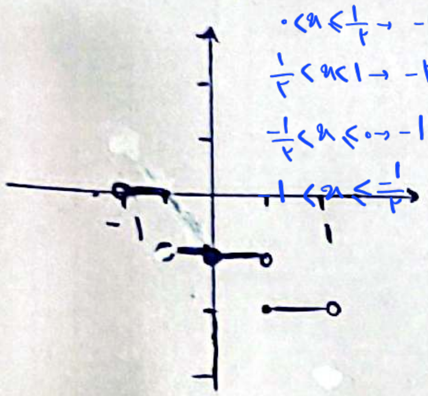
۷ الف) $y = [-2x] - 1 \quad x \in (-1, 1)$

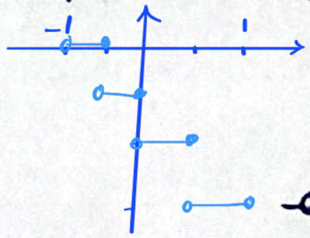
$0 < x \leq \frac{1}{2} \rightarrow -2$

$\frac{1}{2} < x < 1 \rightarrow -3$

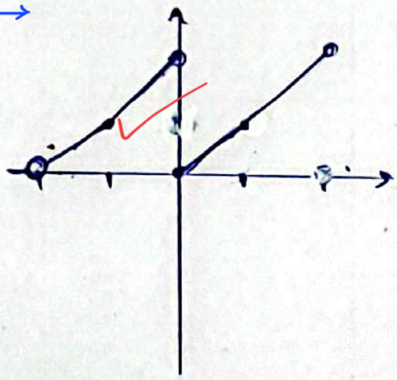
$-\frac{1}{2} < x \leq 0 \rightarrow -1$

$-1 < x \leq -\frac{1}{2}$





ب) $y = x - 2\left[\frac{x}{2}\right] \quad x \in (-2, 2)$



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$a + [b] = 4.2 \Rightarrow [a] + [b] = 4$

$b - [a] = 2.4 \Rightarrow [b] - [a] = 2$

$\Rightarrow \begin{cases} [a] + [b] = 4 \\ [b] - [a] = 2 \end{cases}$

$2[b] = 6 \Rightarrow [b] = 3 \Rightarrow [a] = 1$

$a + 3 = 4.2 \Rightarrow a = 1.2$

$b - [a] = b - 1 = 2.4 \Rightarrow b = 3.4$

$a + b = 1.2 + 3.4 = 4.6$

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$(1+\sqrt{2})^6 + (1-\sqrt{2})^6 = 198 \Rightarrow (1+\sqrt{2})^6 = 198 - (1-\sqrt{2})^6 \overset{\star}{\Longrightarrow} [(1+\sqrt{2})^6] = 197$

$\star : 0 < (1-\sqrt{2})^6 < 1$

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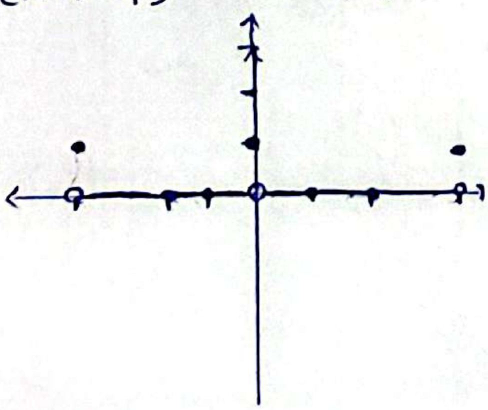
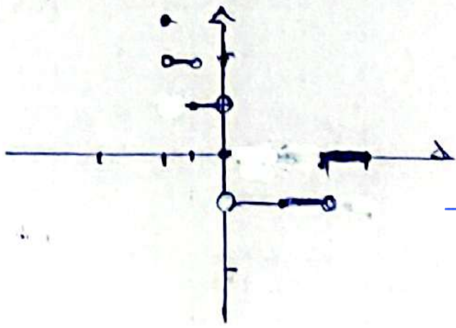
۸ الف) $y = [\,||x| - 1|\,] \quad x \in [-2, 2]$



ب) $y = [x^2 - 2x] \quad x \in [-1, 2]$



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