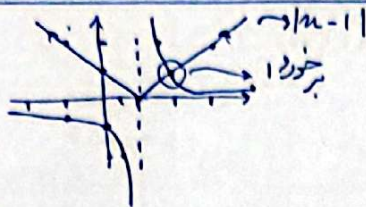


$$\frac{1}{n-1} = |n-1|$$



$$\frac{1}{n-1} = \frac{n-1}{(n-1)^2} = \frac{n-1}{n^2-2n+1}$$

ارکمان دارد

$$\left| \frac{r_{n-1}}{n} \right| < 2 \Rightarrow \begin{cases} \frac{r_{n-1}}{n} < 2 \Rightarrow \frac{r_{n-1}-r_n}{n} < 0 \Rightarrow \frac{-1}{n} < 0 \Rightarrow \frac{0}{+\infty} - \frac{0}{+\infty} = \text{red}^0, +\infty \text{ (1)} \\ -2 < \frac{r_{n-1}}{n} \Rightarrow 0 < \frac{r_{n-1}+r_n}{n} \Rightarrow 0 < \frac{r_{n-1}}{n} \Rightarrow \frac{0}{+\infty} - \frac{1}{+\infty} \Rightarrow n \in (-\infty, 0) \cup \left(\frac{1}{2}, +\infty\right) \end{cases}$$

$$\left| \frac{n-2}{r_{n+1}} \right| > 1 \Rightarrow \begin{cases} \frac{n-2}{r_{n+1}} > 1 \Rightarrow \frac{n-2-r_{n+1}}{r_{n+1}} > 0 \Rightarrow \frac{2-n}{r_{n+1}} > 0 \Rightarrow \frac{-2}{-\frac{1}{2}+\frac{1}{2}} = n \in (-2, -\frac{1}{2}) \text{ (2)} \\ \frac{n-2}{r_{n+1}} < -1 \Rightarrow \frac{n-2+r_{n+1}}{r_{n+1}} < 0 \Rightarrow \frac{r_{n+1}-1}{r_{n+1}} < 0 \Rightarrow \frac{-\frac{1}{2}}{+\frac{1}{2}-\frac{1}{2}} = n \in \left(-\frac{1}{2}, \frac{1}{2}\right) \end{cases}$$

$$(1) \cap (2) \Rightarrow n \in \left(-\frac{1}{2}, -\frac{1}{2}\right) \Rightarrow n = -\frac{1}{2}$$

$$\begin{aligned} n^2 < |n+9|, |n-a| < \beta & \quad |n-a| < \beta \Rightarrow \begin{cases} n-a < \beta \Rightarrow n < \beta+a \\ -\beta < n-a \Rightarrow n > a-\beta \\ a-\beta < n < \beta+a \end{cases} \\ n^2 < n+9 \Rightarrow n^2-n-9 < 0 \Rightarrow \frac{-1}{+\frac{1}{2}-\frac{1}{2}} = -2 < n < 3 \\ n+9 < -n^2 \Rightarrow n^2+n+9 < 0 & \quad \begin{cases} \alpha-\beta = -2 \\ \alpha+\beta = 3 \\ \alpha \geq 1 \Rightarrow \alpha = \frac{1}{2}, \beta = \frac{5}{2} \end{cases} \end{aligned}$$

$$\begin{aligned} y &= |n+1| - |r_n| + |n-2| \\ \max: |n+1| - |r_n| + |n-2| & \Rightarrow n=0 \Rightarrow |1| - |0| + |-2| = 3 \checkmark \Rightarrow \max \\ \min: 0 - \max 0 & \Rightarrow \frac{|n+1|}{n=-1} - |r_n| + \frac{|n-2|}{n=1} \Rightarrow \begin{cases} n=1: |2| - |1| + |0| = 3-1=2 \checkmark \min \\ n=-1: |0| - |-2| + |-3| = -2+3=1 \end{cases} \\ \min, \max &= -1 + 3 = 2 \end{aligned}$$

$$\begin{aligned} y &= \left| \frac{1}{2}n \right| - 2 \xrightarrow{\text{مشتق}} y = \left| \frac{1}{2}n + 1 \right| - 2 \xrightarrow{\text{مشتق}} y = \left| \frac{1}{2}n + 1 \right| - 1 \\ \left| \frac{1}{2}n \right| - 2 &= \left| \frac{1}{2}n + 1 \right| - 1 \Rightarrow \underbrace{\left| \frac{1}{2}n \right|}_0 - \underbrace{\left| \frac{1}{2}n + 1 \right|}_{-1} = 1 \Rightarrow \frac{-1}{\frac{1}{2}n + \frac{1}{2}n} - \frac{0}{-\frac{1}{2}n - \frac{1}{2}n} = \frac{1}{\frac{1}{2}n - \frac{1}{2}n} \\ -n - 1 &\geq 1 \Rightarrow -n \geq 2 \Rightarrow n = -2 \end{aligned}$$

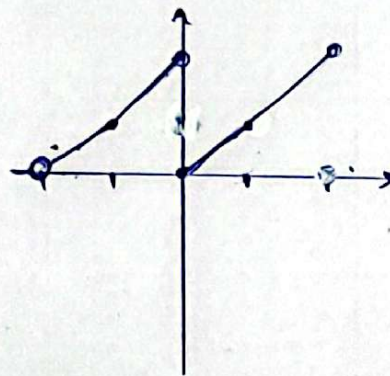
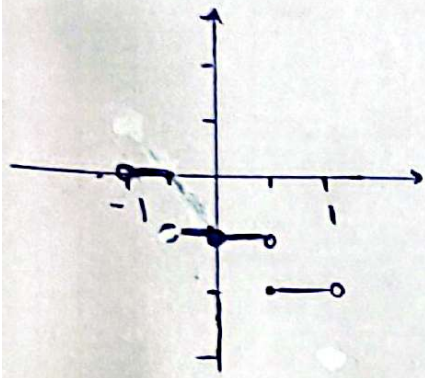
الف) $f(n) = [1-n] - [n+1] \Rightarrow \frac{-1}{1+n+n+1} \frac{1}{1+n-n-1} \Rightarrow R_f = \mathbb{Z} - (-\infty, +\infty)$ (9)

$\frac{-1}{1+n+n+1} \frac{1}{1+n-n-1} \Rightarrow \frac{-1}{2n+2} \frac{1}{-n-1} \Rightarrow \frac{1}{n+1} \frac{1}{-n-1} \Rightarrow \frac{1}{-n^2-2n-1} \Rightarrow \frac{1}{-(n+1)^2}$

ب) $f(n) = \frac{1}{n} - \left[\frac{n+1}{n} \right] \Rightarrow \begin{cases} \frac{1}{n} - 1 & n > 0 \Rightarrow R_f = [0, +\infty) \textcircled{1} \\ \frac{1}{n} + 1 & n < 0 \Rightarrow R_f = (-\infty, 0] \textcircled{2} \end{cases} \Rightarrow R_f = \mathbb{R}$

$\ominus -1, 1, \oplus$

الف) $y = [-x] - 1 \quad x \in (-1, 1)$ ب) $y = x - 2[x] \quad x \in (-2, 2)$ (10)



$a + [b] = r, r \Rightarrow [a] + [b] = r \Rightarrow \begin{cases} [a] + [b] = r \\ [b] - [a] = r \end{cases}$ (11)

$b - [a] = r, r \Rightarrow [b] - [a] = r$

$a + r = r, r \Rightarrow a = 1, r$

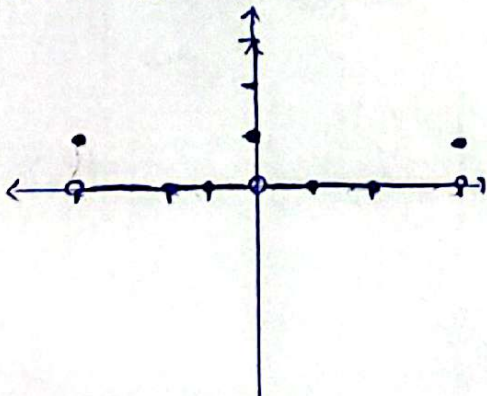
$b - [a] = b - 1 = r, r \Rightarrow b = r + 1, r$

$a + b = 1, r + r + 1 = r, r$

$(1+\sqrt{2})^9 + (1-\sqrt{2})^9 = 198 \Rightarrow (1+\sqrt{2})^9 = 198 - (1-\sqrt{2})^9 \xrightarrow{*} [(1+\sqrt{2})^9] = 198$ (12)

$\star \Rightarrow 0 < (1-\sqrt{2})^9 < 1$

الف) $y = [|n| - 1] \quad x \in [-2, 2]$



ب) $y = [x^2 - x] \quad x \in [-1, 2]$ (13)

