نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....A.....

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۱) $\frac{1}{x-1} = |x-1| \xrightarrow{x-1=t} \frac{1}{t} = |t| \to \frac{1}{t^2} = t^2 \Rightarrow t^2 - \frac{1}{t^2} = 0 \Rightarrow t^4 - 1 = 0$

$\Rightarrow (t^2-1)(t^2+1) \Rightarrow ((x-1)-1)((x-1)+1)(x^2-2x+2) = 0$

$(t-1)(t+1)$; $x-2$; x ; $\Delta<0$

$x-2=0 \Rightarrow x=2$; $x=0$

$\Rightarrow x=0 \Rightarrow \frac{1}{-1} \neq |0-1|$ غ ق ق

$\Rightarrow x=2 \Rightarrow \frac{1}{1} = |2-1| \to$ ق ق $\Rightarrow$ یک ریشه دارد ✓

۲) الف) $\left|\frac{2x-1}{x}\right| < 2 \Rightarrow -2 < \frac{2x-1}{x} < 2 \to \frac{2x-1}{x} < 2 \to \frac{-1}{x} < 0 \Rightarrow \frac{1}{x} > 0$; $(0, +\infty)$

$\frac{2x-1}{x} > -2 \to \frac{4x-1}{x} > 0$; $(-\infty, 0) \cup (\frac{1}{4}, +\infty)$

$\Rightarrow D = (\frac{1}{4}, +\infty)$

ب) $\left|\frac{x-2}{2x+1}\right| > 1 \Rightarrow 1 < \frac{x-2}{2x+1} < -1 \to \frac{x-2}{2x+1} > 1 \Rightarrow \frac{-x-3}{2x+1} > 0$; $(-3, -\frac{1}{2})$

$\Rightarrow \frac{x-2}{2x+1} < -1 \Rightarrow \frac{3x-1}{2x+1} < 0$; $(-\frac{1}{2}, \frac{1}{3})$

$\Rightarrow D = (-3, \frac{1}{3}) - \{-\frac{1}{2}\}$

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۳) $x^2 < |x+6| \to -x-6 < x^2 < x+6$

$x^2 < x+6 \to x^2 - x - 6 < 0 \to \Delta = 25 \Rightarrow \frac{1 \pm 5}{2} \to 3, -2$; $(-2, 3)$

$x^2 > -x-6 \to x^2 + x + 6 > 0 \to \Delta < 0$ ✓

$\Rightarrow -\beta < x - \alpha < \beta \Rightarrow \underbrace{\alpha-\beta}_{-2} < x < \underbrace{\beta+\alpha}_{3} \Rightarrow \begin{cases} \alpha-\beta=-2 \\ \alpha+\beta=3 \end{cases}$

$2\alpha = 1 \Rightarrow \alpha = \frac{1}{2}$

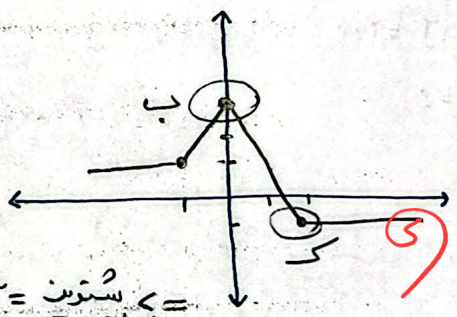
۴) $y = \underbrace{|x+1|}_{x=-1} - \underbrace{|2x|}_{x=0} + \underbrace{|x-2|}_{x=2}$

$\to -x-1+2x-x+2 = 1 \to x < -1$

$\to x+1+2x-x+2 = 2x+4 \to -1 \leq x \leq 0$

$\to x+1-2x-x+2 = -2x+3 \to 0 < x \leq 2$

$\to x+1-2x+x-2 = -1 \to x > 2$



$\Rightarrow$ بیشترین $= 3$; کمترین $= -1$

$\Rightarrow 3 - (-1) = 4$

۵) $y = |\frac{1}{2}x| - 2$

$\hookrightarrow y' = |\frac{1}{2}(x+4)| - 2 + 1 = |\frac{1}{2}x + 2| - 1$

$y = y' \Rightarrow |\frac{1}{2}x| - 2 = |\frac{1}{2}x + 2| - 1$

$|\frac{1}{2}x| = |\frac{1}{2}x + 2| + 1 \Rightarrow |x| = |x+4| + 2$

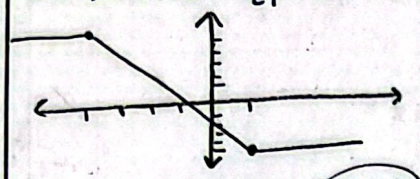
$\to x = x+4 \to$ ✗

$\to -x = x+4 \to x = -2$ ✓ $-4 < x < 0$

$x = -x-2 = 2$ ✓ $x < -4$

$\Rightarrow \boxed{x = -2}$

الف) $F(x) = [|1-x| - |x+1|] \Rightarrow x=1 \ , \ x=-1$



$\hookrightarrow R = [-5 \le y < 5] \Rightarrow R = \{-5, -4, \dots, 4, 5\}$

1,5

ب) $F(x) = \frac{1}{x} - \left[\frac{x+1}{x}\right] = -1 + \underbrace{\frac{1}{x} - \left[\frac{1}{x}\right]}_{0 \le y < 1} \Rightarrow R = [-1, 0)$

$1 + \left[\frac{1}{x}\right]$

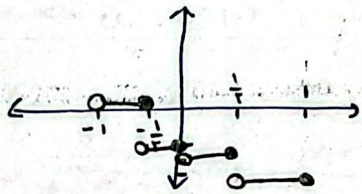
الف) $y = [-2x] - 1 \quad x \in (-1, 1)$

$-1 < x \le -\frac{1}{2} \Rightarrow [-2x] - 1 = 0$

$-\frac{1}{2} < x < 0 \Rightarrow [-2x] - 1 = -1$

$0 < x \le \frac{1}{2} \Rightarrow [-2x] - 1 = -2$

$\frac{1}{2} \le x < 1 \Rightarrow [-2x] - 1 = -3$



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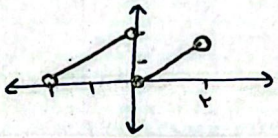
ب) $x - 2\left[\frac{x}{2}\right] \quad x \in (-2, 2)$

$-2 < x < 0 \rightarrow x+2$

$0 \le x < 2 \rightarrow x$



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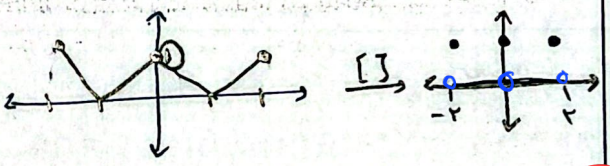
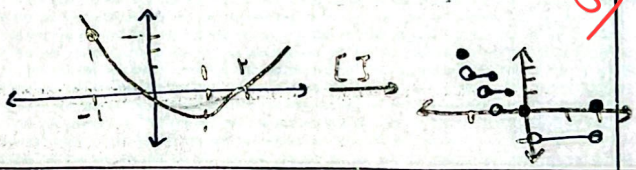
الف) $y = [|2x| - 1] \quad x \in [-2, 2]$



ب) $y = [x^2 - 2x] \quad x \in [-1, 2]$



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$a + [b] = 4,2 \rightarrow a = 0,2 \Rightarrow [a] + [b] = 4$

$b - [a] = 2,4 \rightarrow [b] - [a] = 2 \Rightarrow b - 1 = 2,4 \Rightarrow b = 3,4$

$a + b = ?$

$\Rightarrow \begin{cases} [a] + [b] = 4 \\ \ominus \ (-[a] + [b] = 2) \end{cases} \Rightarrow 2[a] = 2 \Rightarrow [a] = 1 \rightarrow a = 1,2$

$a + b = 4,6$

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$(1+\sqrt{2})^4 + (1-\sqrt{2})^4 = 198$

$(1-1,4)^2$ ① $\simeq 1,4$

$(1+\sqrt{2})^4 = ? \Rightarrow 197,\_\_\_ \Rightarrow \underbrace{[(1+\sqrt{2})^4]}_{197,\_\_\_} = 197$

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$(1+\sqrt{2}) = -1 \le x < 0$

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