

$$\frac{1}{x-1} = |n-1| \quad n > 1 \rightarrow n^2 - 2n + 1 = 1 \Rightarrow n^2 - 2n = 0 \quad \begin{cases} n=0 \\ n=2 \end{cases}$$

$$n=1 \rightarrow 0 \text{ و } 1 \text{ و } 2$$

$$n < 1 \rightarrow -n^2 + 2n - 1 = 1 \Rightarrow -n^2 + 2n - 2 = 0 \Rightarrow \Delta < 0 \quad \text{ریشه ندارد}$$

معادله یک ریشه دارد (x=2)

$$\text{الف) } \left| \frac{r_{n-1}}{x} \right| < 2 \rightarrow -2 < \frac{r_{n-1}}{x} < 2 \quad \frac{r_{n-1}}{x} \cdot (x=2) > 0 \quad \text{I} \quad \left\{ \begin{array}{l} \text{II} \\ \text{III} \end{array} \right\} \rightarrow x \in \left(\frac{1}{2}, +\infty \right)$$

$$\text{ب) } \left| \frac{n-2}{r_{n+1}} \right| > 1 \rightarrow \frac{n-2}{r_{n+1}} > 1 \Rightarrow \frac{n-2}{r_{n+1}} > 0 \Rightarrow \frac{-2}{-1} + \frac{1}{1} = -1 \rightarrow x \in (-2, -\frac{1}{2})$$

$$\frac{n-2}{r_{n+1}} < -1 \Rightarrow \frac{r_{n-1}}{r_{n+1}} < 0 \Rightarrow \frac{-1}{+1} - \frac{1}{1} = -2 \rightarrow x \in (-\frac{1}{2}, \frac{1}{2})$$

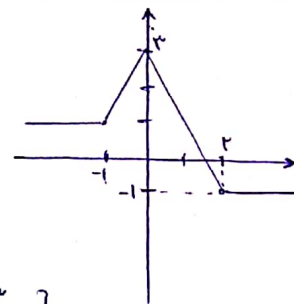
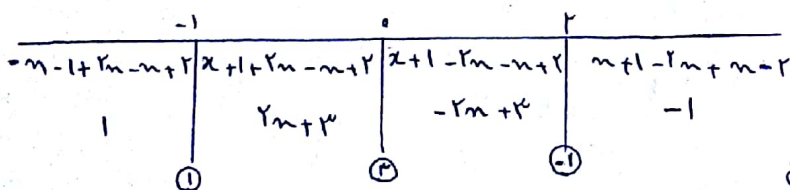
$$\rightarrow x \in (-2, -\frac{1}{2}) \cup (-\frac{1}{2}, \frac{1}{2})$$

$$x^2 < |n+4| \rightarrow x > -2 \rightarrow x^2 - x - 4 < 0 \Rightarrow (n-3)(n+2) < 0 \quad \frac{-r}{+4} - \frac{r}{-4} = -1 \Rightarrow x \in (-2, 2)$$

$$n < -2 \rightarrow x^2 < 4 - n \Rightarrow n^2 + x + 4 < 0 \Rightarrow \Delta < 0$$

$$x \in (-2, 2) \quad \frac{2-2}{2} = \frac{1}{2} \rightarrow \frac{-\infty}{2} < n - \frac{1}{2} < \frac{\infty}{2} \Rightarrow |n - \frac{1}{2}| < \frac{\infty}{2} \quad \boxed{x = \frac{1}{2}}$$

$$y = |n+1| - |n| + |n-2|$$



$$\left. \begin{array}{l} y_{\max} = 3 \\ y_{\min} = -1 \end{array} \right\} 3 + (-1) = 2$$

$$y = \left| \frac{1}{2}n \right| - 2 \rightarrow y = \left| \frac{1}{2}(n+4) \right| - 2 \rightarrow y = \left| \frac{1}{2}(n+4) \right| - 1 \Rightarrow y = \left| \frac{1}{2}n + 2 \right| - 1$$

$$\text{قطع ریشه} \rightarrow \left| \frac{1}{2}n \right| - 2 = \left| \frac{1}{2}n + 2 \right| - 1 \xrightarrow{x^2} |x| - 4 = |n+4| - 2 \Rightarrow |n| + |n+4| = 2$$

$$n > 0 \rightarrow \frac{1}{2}n = \frac{1}{2}n + 2 \quad \text{قطع}$$

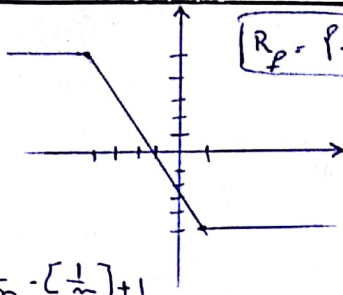
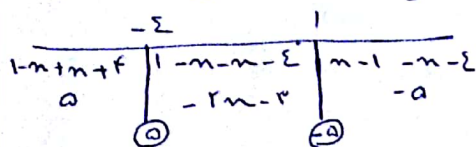
$$-2 < n < 0 \rightarrow -\frac{1}{2}n - 2 = \frac{1}{2}n + 2 \Rightarrow \boxed{n = -4}$$

$$n < -2 \rightarrow \frac{1}{2}n - 2 = -\frac{1}{2}n - 2 \quad \text{قطع}$$

$$n = 0 \rightarrow -2 \neq 2 - 1 \quad \text{قطع} \quad n = -2 \rightarrow 0 \neq -1 \quad \text{قطع}$$

قطع $x = 3$ و $x = -1$

$$i) f(n) = \lfloor (1-n) - \lfloor n+1 \rfloor \rfloor$$



$$R_f = \{-\infty, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, \infty\}$$

$$ii) f(n) = \frac{1}{n} - \lfloor \frac{n+1}{n} \rfloor \rightarrow f(n) = \frac{1}{n} - \lfloor \frac{1}{n} \rfloor + 1$$

$$\frac{1}{n} - (\lfloor \frac{1}{n} \rfloor + 1) \rightarrow \frac{1}{n} - \lfloor \frac{1}{n} \rfloor - 1 \rightarrow \begin{cases} \frac{1}{n} \in \mathbb{Z} \rightarrow f(n) = -1 \\ \frac{1}{n} \notin \mathbb{Z} \rightarrow f(n) < -1 \end{cases} \rightarrow R_f = [-1, 0)$$

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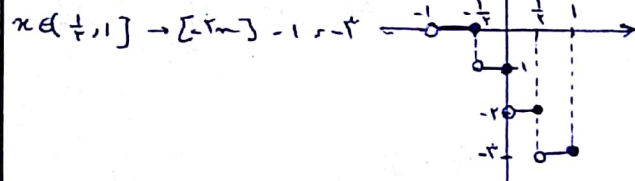
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$$i) y = \lfloor -2n \rfloor - 1, n \in (-1, 1)$$

$$x \in (-1, -\frac{1}{2}] \rightarrow \lfloor -2n \rfloor - 1 = 0$$

$$n \in (-\frac{1}{2}, 0] \rightarrow \lfloor -2n \rfloor - 1 = -1$$

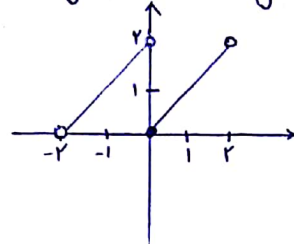
$$x \in (0, \frac{1}{2}] \rightarrow \lfloor -2n \rfloor - 1 = -2$$



$$ii) y = n - 2 \lfloor \frac{n}{2} \rfloor, n \in (-2, 2)$$

$$x \in (-2, 0) \rightarrow y = n - 2 \lfloor -1 \rfloor \rightarrow y = n + 2$$

$$n \in [0, 2) \rightarrow y = n - 2 \lfloor 0 \rfloor \rightarrow y = n$$

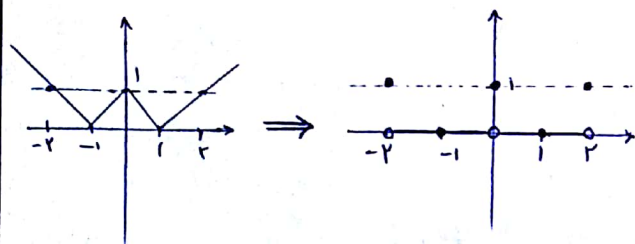


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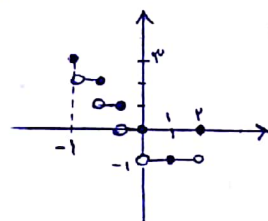
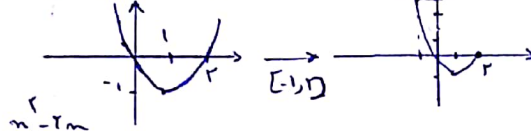
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$$i) y = \lfloor |n| - 1 \rfloor, n \in [-2, 2]$$

$$2-1 \rightarrow |n|-1 \rightarrow \lfloor |n|-1 \rfloor \rightarrow \lfloor |n|-1 \rfloor$$



$$ii) y = \lfloor n^2 - 2n \rfloor, n \in [-1, 2]$$



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$$a + \lfloor b \rfloor = 4, 2 \rightarrow a, 2 \in 1, 2 \Rightarrow a = \lfloor a \rfloor + 1, 2$$

$$b - \lfloor a \rfloor = 1, 4 \rightarrow b, 4 \in 1, 4 \Rightarrow b = \lfloor b \rfloor + 1, 4$$

$$\text{e.g. } \underbrace{a + \lfloor b \rfloor}_{\lfloor a \rfloor + 1, 2} + \underbrace{b - \lfloor a \rfloor}_{\lfloor b \rfloor + 1, 4} = 4, 4 \Rightarrow \lfloor a \rfloor + 1, 2 + \lfloor b \rfloor + 1, 4 - \lfloor a \rfloor = 4, 4 \rightarrow 2 \lfloor b \rfloor = 4 \Rightarrow \lfloor b \rfloor = 2$$

$$b = 3 + 1, 4 = 3, 4 \quad a = 1, 2$$

$$a + b = 3, 4 + 1, 2 = 4, 4$$

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$$(1+\sqrt{2})^4 + (1-\sqrt{2})^4 = 198 \rightarrow (1+\sqrt{2})^4 = 198 - (1-\sqrt{2})^4$$

$$1-\sqrt{2} \approx -0,41 \rightarrow 0 < (1-\sqrt{2})^4 < 1$$

$$(1+\sqrt{2})^4 = 198 - (1-\sqrt{2})^4 = 194, \dots \Rightarrow \lfloor (1+\sqrt{2})^4 \rfloor = 194$$

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