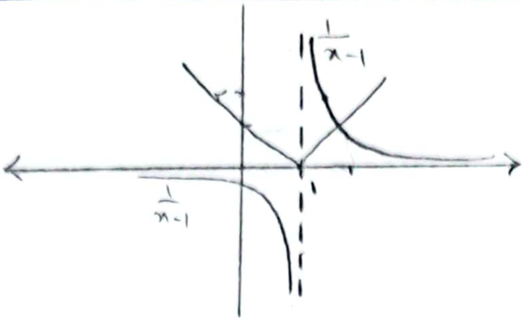
دیا بابائی　　۱۷　　یازدهم ریاضی　A　　تکلیف نهایی ۵ و ۶

۱)

$$\frac{1}{x-1} = |x-1|$$



یک ریشه دارد
چون ۲ نمودار در یک نقطه
همدیگر را قطع می کنند

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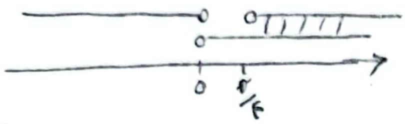
۲)

الف) $\left|\frac{2x-1}{x}\right| < 2$

$$-2 < \frac{2x-1}{x} < 2 \rightarrow \begin{cases} \frac{2x-1}{x} < 2 \rightarrow \frac{\cancel{2x}-1-\cancel{2x}}{x} < 0 \rightarrow \frac{-1}{x} < 0 \rightarrow x > 0 \\ \frac{2x-1}{x} > -2 \rightarrow \frac{2x-1+2x}{x} > 0 \rightarrow \frac{4x-1}{x} > 0 \end{cases}$$

0, $\frac{1}{4}$: $+$ ت $-$ 0 $+$ $\rightarrow (-\infty, 0) \cup (\frac{1}{4}, +\infty)$



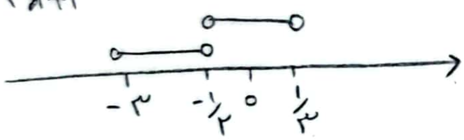
ج: $(\frac{1}{4}, +\infty)$

ب) $\left|\frac{x-2}{2x+1}\right| > 1$

$$\begin{cases} \frac{x-2}{2x+1} > 1 \rightarrow \frac{x-2-2x-1}{2x+1} > 0 \rightarrow \frac{-x-3}{2x+1} > 0 \\ \text{یا} \\ \frac{x-2}{2x+1} < -1 \rightarrow \frac{x-2+2x+1}{2x+1} < 0 \rightarrow \frac{3x-1}{2x+1} < 0 \rightarrow \end{cases}$$

-3, $-\frac{1}{2}$: $-$ 0 $+$ ت $-$ 　 $(-3, -\frac{1}{2})$

$-\frac{1}{2}$, $\frac{1}{3}$: $+$ ت $-$ 0 $+$ 　 ج $(-\frac{1}{2}, \frac{1}{3})$



ج: $(-3, \frac{1}{3}) - \{-\frac{1}{2}\}$

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۳)

$$x^2 < |x+6|$$

$$|x+6| > x^2 \rightarrow \begin{cases} x+6 > x^2 \Rightarrow x^2-x-6 < 0 \rightarrow (x-3)(x+2) < 0 \rightarrow \\ x+6 < -x^2 \rightarrow x^2+x+6 < 0 \rightarrow \Delta < 0 \quad \text{معادله جواب ندارد} \end{cases}$$

-2, 3: $+$ 0 $-$ 0 $+$ 　 $(-2, 3)$

$a > 0 \rightarrow$ معادله همواره مثبت

$$-2 < x < 3 \rightarrow \left|x - \frac{1}{2}\right| < \frac{5}{2}$$

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۴)

$$y = |x+1| - 2|x| + |x-2|$$

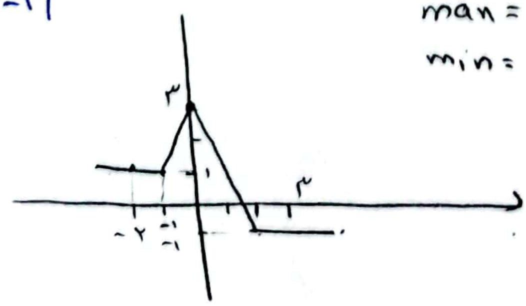
$x = -2 \rightarrow y = 1$

$x = -1 \rightarrow y = 1$

$x = 0 \rightarrow y = 3$

$x = 2 \rightarrow y = -1$

$x = 3 \rightarrow y = -1$



$$\left.\begin{matrix} max = 3 \\ min = -1 \end{matrix}\right\} max + min = 3 + (-1) = 2$$

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$y = \left|\frac{1}{2}x\right| - 2 \xrightarrow{x+4} y = \left|\frac{1}{2}(x+4)\right| - 2 \xrightarrow{y \to y+1} y = \left|\frac{1}{2}x + 2\right| - 2 + 1$

$\frac{1}{2}(x+4) = \frac{1}{2}x+2$

$y = \left|\frac{1}{2}x+2\right| - 1 \implies \frac{1}{2}|x| - 2 = \frac{1}{2}|x+4| - 1 \implies \frac{1}{2}|x| - 2 + 1 = \frac{1}{2}|x+4|$

$\implies \frac{1}{2}|x| - 1 = \frac{1}{2}|x+4| \xrightarrow{\times 2} |x| - 2 = |x+4|$

		-4		0	
x	$-$		$-$	0	$+$
$x+4$	$-$	0	$+$		$+$
	①		②		③

① $x < -4 \implies -x - 2 = -x - 4$ غ ق

② $-4 \leq x \leq 0 \to -x - 2 = x + 4 \to -2x = +6 \to x = -3$ ق ق

$-4 \leq -3 \leq 0$

③ $x > 0 \to x - 2 = x + 4 \to -2 \neq 4$

تنها نقطه برخورد $x = -3$ است.

2

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۶)

الف) $f(x) = [\,|1-x| - |x+4|\,]$ می‌دانیم $|1-x| = |x-1|$

$|x-1| - |x+4| = p(x) \to x = 1 \,,\, x = -4$

		-4		1	
$x-1$	$-$		$-$	0	$+$
$x+4$	$-$	0	$+$		$+$
	①		②		③

① $x < -4$: $-x + 1 + x + 4 = 5$

② $-4 \leq x \leq 1 \to -x + 1 - x - 4 = -2x - 3$

③ $x \geq 1 \to x - 1 - x - 4 = -5$

$$p(x) = \begin{cases} 5 & x < -4 \\ -2x-3 & -4 \leq x \leq 1 \\ -5 & x > 1 \end{cases}$$

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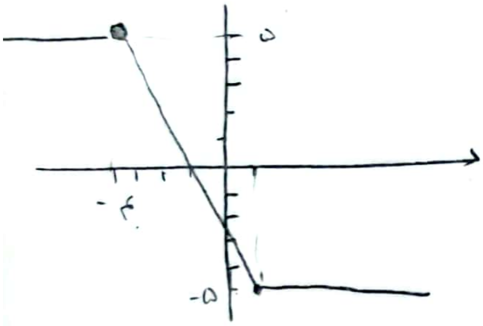

$-5 \leq p(x) \leq 5 \to f(x) = [p(x)] \quad p(x) \in [-5, 5]$

$R_f = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

ب) $f(x) = \frac{1}{x} - \left[\frac{x+1}{x}\right] = \frac{1}{x} - \left[1 + \frac{1}{x}\right] = \frac{1}{x} - \left(\left[\frac{1}{x}\right] + 1\right) = \frac{1}{x} - \left[\frac{1}{x}\right] - 1$

$0 \leq \frac{1}{x} - \left[\frac{1}{x}\right] < 1 \xrightarrow{-1} -1 \leq \frac{1}{x} - \left[\frac{1}{x}\right] - 1 < 0$

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۷)

الف) $y = [-2x] - 1 \qquad -1 < x < 1 \quad - \quad -2 < 2x < 2$

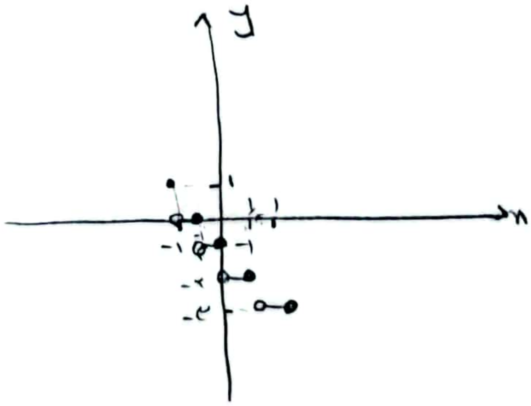
$x = -1 \to [-2x] = 2 \to y = 1$

$-1 < x \leq -\frac{1}{2} \to [-2x] = 1 \to y = 0$

$-\frac{1}{2} < x \leq 0 \to [-2x] = 0 \to y = -1$

$0 < x \leq \frac{1}{2} \to [-2x] = -1 \to y = -2$

$\frac{1}{2} < x < 1 \to [-2x] = -2 \to y = -3$



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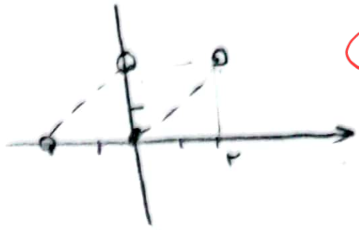
ب) $y = x - 2\left[\frac{x}{2}\right]$ $x \in (-2,2)$ $-1 < \frac{x}{2} < 1$

$-2 < x < 0 \to \left[\frac{x}{2}\right] = -1 \to y = x - 2(-1) = x + 2$

$0 \leq x < 2 \to \left[\frac{x}{2}\right] = 0 \to y = x - 2(0) = x$



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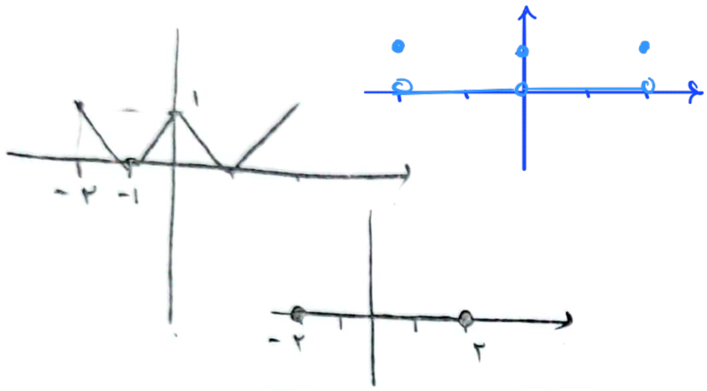
(۱۸

الف) $y = [\,||x|-1|\,]$ $x \in [-2,2]$ / ب) $y = [x^2 - 2x] = [x^2 - 2x + 1 - 1] = [(x-1)^2 - 1]$



$-1 \leq x < 2$

$-1 \leq x < 0$

$0 \leq x < 1$

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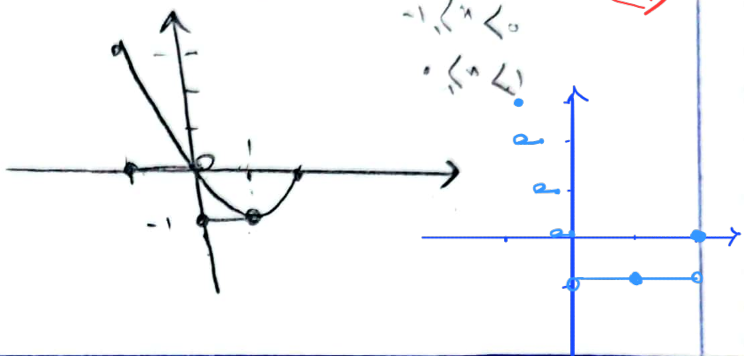

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$$\begin{cases} a + [b] = 4.2 \\ b - [a] = 2.4 \end{cases}$$

$a + [b] + b - [a] = 6.6$

$a + b + [b] - [a] = 6.6 \xrightarrow{+[b]-[b]}$

$0 \leq a - [a] < 1$

$0 \leq b - [b] < 1$

$0 \leq a - [a] + b - [b] < 2$

$\underbrace{a - [a] + b - [b]}_{0} + 2[b] = 6.6 \to 2[b] = 6$

$[b] = 3$

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$a + 3 = 4.2 \to a = 1.2$, $b - [a] = 2.4$

$a + b = 1.2 + 3.4 = 4.6$

$b - 1 = 2.4$

$b = 3.4$

(۲۰

$(1+\sqrt{2})^4 + \underset{(\sqrt{2}-1)^4}{(1-\sqrt{2})^4} = 198 \to [(1+\sqrt{2})^4]$

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$1 - \sqrt{2} = \frac{-1}{1+\sqrt{2}} \to x_1 = \sqrt{2}+1 , x_2 = \sqrt{2}-1 \to x^2 - 2\sqrt{2}x + 1 = 0$

$0 < (1-\sqrt{2})^4 < 1$ $[(1+\sqrt{2})^4] = [198 - (1-\sqrt{2})^4]$

$= [(1+\sqrt{2})^4] = 198 + [\underbrace{-(1-\sqrt{2})^4}_{0^-}] = 198 - 1 = 197$