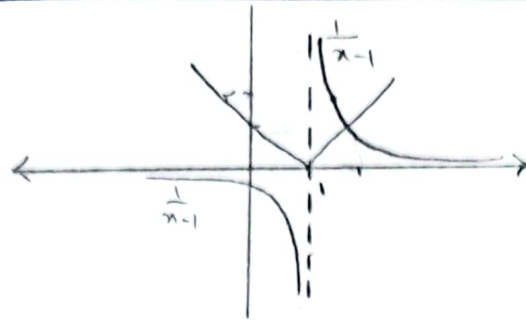


$$\frac{1}{x-1} = |x-1|$$



یک ریشه دارد
چون ۲ نمودار در یک نقطه
همگرز قطع می کنند

(۱)

$$\text{الف) } \left| \frac{2x-1}{x} \right| < 2$$

$$-2 < \frac{2x-1}{x} < 2 \rightarrow \begin{cases} \frac{2x-1}{x} < 2 \rightarrow \frac{2x-1-2x}{x} < 0 \rightarrow \frac{-1}{x} < 0 \rightarrow x > 0 \\ \frac{2x-1}{x} > -2 \rightarrow \frac{2x-1+2x}{x} > 0 \rightarrow \frac{4x-1}{x} > 0 \end{cases}$$

$$| + \frac{0}{x} - \frac{1}{x} + \frac{1}{x} | \rightarrow (-\infty, 0) \cup (\frac{1}{4}, \infty)$$



$$\text{ج: } (\frac{1}{4}, +\infty)$$

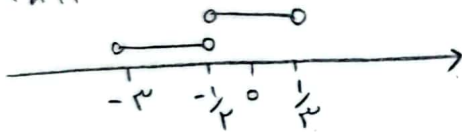
$$\text{ب) } \left| \frac{x-2}{2x+1} \right| > 1$$

$$\begin{cases} \frac{x-2}{2x+1} > 1 \rightarrow \frac{x-2-2x-1}{2x+1} > 0 \rightarrow \frac{-x-3}{2x+1} > 0 \\ \frac{x-2}{2x+1} < -1 \rightarrow \frac{x-2+2x+1}{2x+1} < 0 \rightarrow \frac{3x-1}{2x+1} < 0 \end{cases}$$

$$| -\frac{3}{x} + \frac{1}{x} | - (-\frac{3}{x} + \frac{1}{x})$$

$$\frac{x-2}{2x+1} < -1 \rightarrow \frac{x-2+2x+1}{2x+1} < 0 \rightarrow \frac{3x-1}{2x+1} < 0 \rightarrow | + \frac{1}{x} - \frac{1}{x} + \frac{1}{x} |$$

$$(-\frac{1}{2}, \frac{1}{3})$$



$$\text{ج: } (-\infty, -3) \cup (-\frac{1}{2}, \frac{1}{3})$$

$$x^2 < |x+4|$$

$$|x+4| > x^2 \rightarrow \begin{cases} x+4 > x^2 \Rightarrow x^2-x-4 < 0 \rightarrow (x-3)(x+2) < 0 \rightarrow \frac{-2}{x} - \frac{3}{x} + \frac{3}{x} \\ x+4 < -x^2 \Rightarrow x^2+x+4 < 0 \rightarrow \Delta < 0 \end{cases}$$

$$(-2, 3)$$

معادله جواب ندارد
 $\Delta > 0 \rightarrow$ معادله جواب دارد

$$-2 < x < 3 \rightarrow |x - \frac{1}{x}| < \frac{5}{2}$$

$$y = |x+1| - 2|x| + |x-2|$$

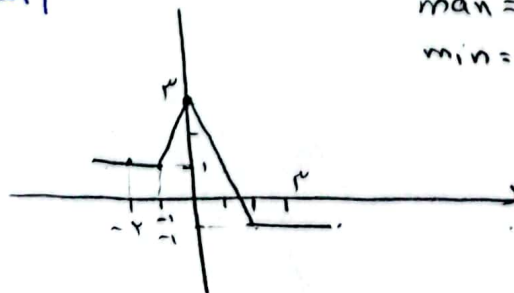
$$x = -2 \rightarrow y = 1$$

$$x = -1 \rightarrow y = 1$$

$$x = 0 \rightarrow y = 3$$

$$x = 2 \rightarrow y = -1$$

$$x = 3 \rightarrow y = -1$$



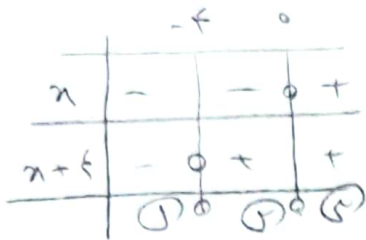
$$\begin{cases} \max = 3 \\ \min = -1 \end{cases} \rightarrow \max + \min = 3 + (-1) = 2$$

(۹)

$$y = \left| \frac{1}{4}x \right| - 2 \xrightarrow{x+4} y = \left| \frac{1}{4}(x+4) \right| - 2 \xrightarrow{y \rightarrow y+1} y = \left| \frac{1}{4}x + 1 \right| - 2 + 1$$

$$y = \left| \frac{1}{4}x + 1 \right| - 1 \Rightarrow \frac{1}{4}|x| - 2 = \frac{1}{4}|x+4| - 1 \Rightarrow \frac{1}{4}|x| - 2 + 1 = \frac{1}{4}|x+4|$$

$$\Rightarrow \frac{1}{4}|x| - 1 = \frac{1}{4}|x+4| \xrightarrow{\times 4} |x| - 4 = |x+4|$$



$$\textcircled{1} x < -4 \Rightarrow -x - 4 = -x - 4 \quad \text{Q.E.D.}$$

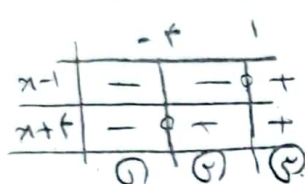
$$\textcircled{2} -4 < x < 0 \Rightarrow -x - 4 = x + 4 \Rightarrow -2x = 8 \Rightarrow x = -4 \quad \text{Q.E.D.}$$

$$x > 0 \Rightarrow x - 4 = x + 4 \Rightarrow -4 \neq 4$$

نتيجة برضو $x = -4$ است

$$\text{الف) } f(x) = \left[|1-x| - |x+4| \right] \quad \text{نلاحظ } |1-x| = |x-1|$$

$$|x-1| - |x+4| = p(x) \rightarrow x=1, x=-4$$

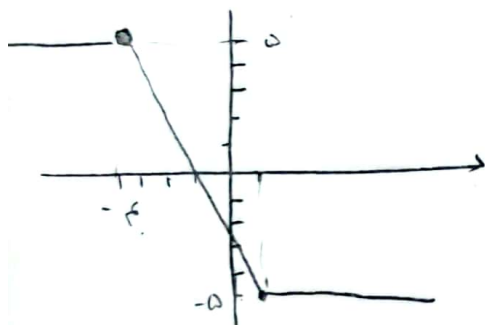


$$\textcircled{1} x < -4 \Rightarrow -x+1-x-4 = -2x-3 = 0$$

$$\textcircled{2} -4 < x < 1 \Rightarrow -x+1-x-4 = -2x-3 = 0$$

$$\textcircled{3} x > 1 \Rightarrow x-1-x-4 = -5 = 0$$

$$p(x) = \begin{cases} 0 & x < -4 \\ -2x-3 & -4 < x < 1 \\ -5 & x > 1 \end{cases}$$



$$-5 \leq p(x) \leq 0 \rightarrow f(x) = [p(x)] \quad p(x) \in [-5, 0]$$

$$R_f = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$$

$$\text{ب) } f(x) = \frac{1}{x} - \left[\frac{x+1}{x} \right] = \frac{1}{x} - \left[1 + \frac{1}{x} \right] = \frac{1}{x} - \left(\left[\frac{1}{x} \right] + 1 \right) = \frac{1}{x} - \left[\frac{1}{x} \right] - 1$$

$$0 \leq \frac{1}{x} - \left[\frac{1}{x} \right] < 1 \xrightarrow{-1} -1 \leq \frac{1}{x} - \left[\frac{1}{x} \right] - 1 < 0$$

$$\text{الف) } y = [-2x] - 1 \quad -1 < x < 1 \quad -2 < 2x < 2$$

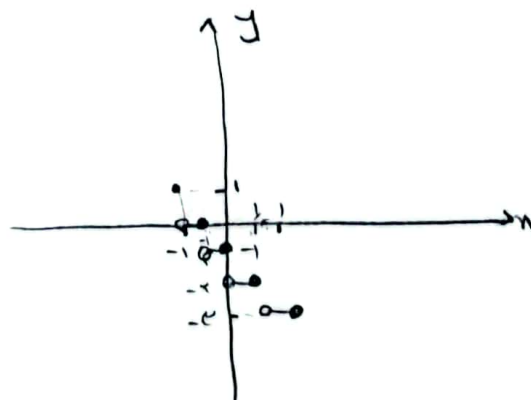
$$x = -1 \rightarrow [-2x] = 2 \rightarrow y = 1$$

$$-1 < x \leq -\frac{1}{2} \rightarrow [-2x] = 1 \rightarrow y = 0$$

$$-\frac{1}{2} < x \leq 0 \rightarrow [-2x] = 0 \rightarrow y = -1$$

$$0 < x \leq \frac{1}{2} \rightarrow [-2x] = -1 \rightarrow y = -2$$

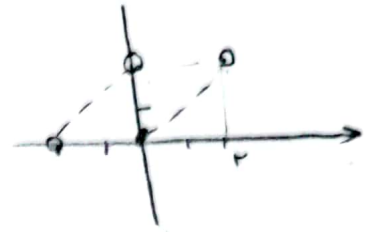
$$\frac{1}{2} < x < 1 \rightarrow [-2x] = -2 \rightarrow y = -3$$



$$-1) y = x - 2 \left[\frac{x}{2} \right] \quad x \in (-2, 2) \quad -1 < \frac{x}{2} < 1$$

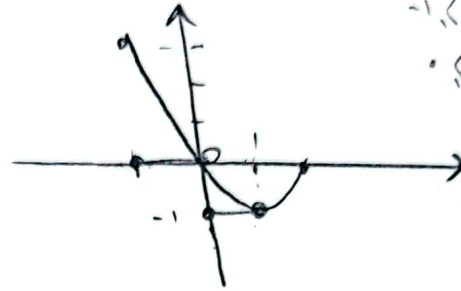
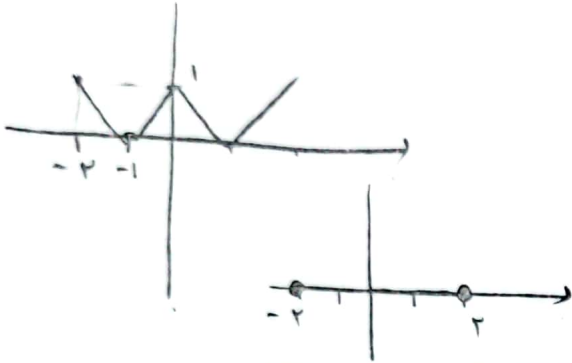
$$-2 < x < 0 \rightarrow \left[\frac{x}{2} \right] = -1 \rightarrow y = x - 2(-1) = x + 2$$

$$0 < x < 2 \rightarrow \left[\frac{x}{2} \right] = 0 \rightarrow y = x - 2(0) = x$$



$$\text{ع) } y = [|x| - 1] \quad x \in [-2, 2] \quad / \quad -1) y = [x^2 - 2x] = [x^2 - 2x + 1 - 1] = [(x-1)^2 - 1]$$

$$\begin{aligned} -1 &\leq x < 2 \\ -1 &< x < 0 \\ 0 &\leq x < 1 \end{aligned}$$



$$\begin{cases} a + [b] = 4, 2 \\ b - [a] = 2, 4 \end{cases}$$

$$a + [b] + b - [a] = 4, 4$$

$$a + b + [b] - [a] = 4, 4 \quad \xrightarrow{+[b] - [b]}$$

$$0 \leq a - [a] < 1$$

$$0 \leq b - [b] < 1$$

$$0 \leq a - [a] + b - [b] < 2$$

$$a - [a] + b - [b] + 2[b] = 4, 4 \rightarrow 2[b] = 4$$

$$[b] = 2$$

$$a + 2 = 4, 2 \rightarrow a = 1, 2$$

$$a + b = 1, 2 + 2, 4 = 4, 4$$

$$b - [a] = 2, 4$$

$$b - 1 = 2, 4$$

$$b = 3, 5$$

$$(1 + \sqrt{2})^4 + \frac{(1 - \sqrt{2})^4}{(\sqrt{2} - 1)^4} = 19 \wedge \rightarrow [(1 + \sqrt{2})^4]$$

$$1 - \sqrt{2} = \frac{-1}{1 + \sqrt{2}}$$

$$\rightarrow x_1 = \sqrt{2} + 1, x_2 = \sqrt{2} - 1 \rightarrow x^2 - 2\sqrt{2}x + 1 = 0$$