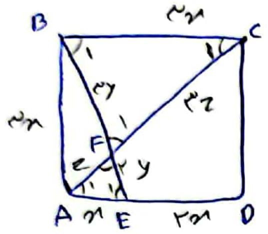
(۱)



$ED = 3AE$

$\frac{EF}{AF} = \frac{y}{z}$

$BC \parallel AD \begin{cases} \hat{B}_1 = \hat{E}_1 \\ \hat{A}_1 = \hat{C}_1 \end{cases}$

متقابل به رأس $\hat{F}_1 = \hat{F}_2$

$\Rightarrow \overset{\triangle}{AEF} \sim \overset{\triangle}{BCF} \Rightarrow \frac{BF}{FE} = \frac{CF}{AF} = \frac{CB}{EA} = \frac{3x}{x} = 3$

$CF = 3z$, $BF = 3y$; $EF = y$, $AF = z$

$\overset{\triangle}{ABC} \Rightarrow AC = 4z \Rightarrow \underbrace{(BC)^2}_{9x^2} + \underbrace{(AB)^2}_{9x^2} = \underbrace{(AC)^2}_{16z^2} \Rightarrow 16z^2 = 18x^2$

$\overset{\triangle}{ABE} \Rightarrow BE = 4y \Rightarrow \underbrace{(AE)^2}_{x^2} + \underbrace{(AB)^2}_{9x^2} = \underbrace{(BE)^2}_{16y^2} \Rightarrow 16y^2 = 10x^2$

$\frac{16y^2}{16z^2} = \frac{10x^2}{18x^2} \Rightarrow \frac{y}{z} = \sqrt{\frac{10}{18}}$

$\frac{\sqrt{5}}{3} = \frac{EF}{AF} = \frac{y}{z}$

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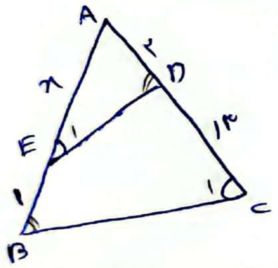
(۲)



$A\hat{E}D = A\hat{C}B$

$x = ?$ ۵

$\begin{rcases} \hat{A} = \hat{A} \\ \hat{E}_1 = \hat{C}_1 \end{rcases} \Rightarrow \overset{\triangle}{AED} \sim \overset{\triangle}{ABC} \Rightarrow \frac{2}{x+1} = \frac{x}{15} \Rightarrow x^2 + x = 30$

$x^2 + x - 30 = 0$

$(x+6)(x-5) = 0$ ⟶ $x = -6$ غ ق ق ، $x = 5$ ✓✓

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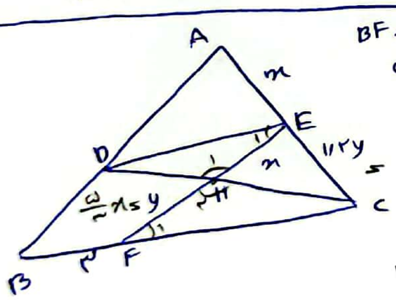
(۳)



$BF = 3$

$3y = 5x \Rightarrow y = \frac{5}{3}x$

$DE \parallel BC$

$BC = ?$

$1/2y = \frac{12}{10} \times \frac{5}{3}x = 2x$

متقابل به رأس $\hat{H}_1 = \hat{H}_2$

$DE \parallel BC \Longrightarrow DE \parallel FC \Rightarrow \hat{E}_1 = \hat{F}_1$ (BC بر FC منطبق است)

$\Rightarrow \overset{\triangle}{DHE} \sim \overset{\triangle}{FHC}$

$\frac{DE}{FC} = \frac{x}{y} = \frac{x}{\frac{5}{3}x} = \frac{3}{5}$ ⟹ (I) $\frac{DE}{FC} = \frac{3}{5}$

$DE \parallel BC \Rightarrow \frac{DE}{BC} = \frac{AE}{AC} \Rightarrow \frac{DE}{\underbrace{BF}_{3}+FC} = \frac{x}{\underbrace{x+1/2y}_{3x}} = \frac{1}{3} \overset{(I)}{\Longrightarrow} \frac{\frac{3}{5}FC}{3+FC} = \frac{1}{3}$

$3 + FC = \frac{9}{5}FC \Rightarrow \frac{4}{5}FC = 3 \Rightarrow FC = \frac{15}{4} \Rightarrow BC = \underbrace{BF}_{3} + \underbrace{FC}_{\frac{15}{4}} \Rightarrow BC = \frac{27}{4}$

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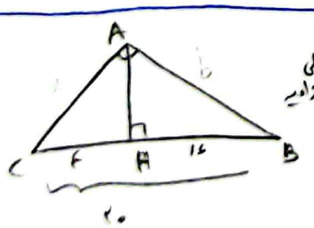
(۴)



طبق روابط طولی در مثلث قائم الزاویه $\Rightarrow$

$\begin{rcases} AC^2 = \underbrace{CH}_{4} \times \underbrace{CB}_{20} \\ AB^2 = \underbrace{BH}_{16} \times \underbrace{CB}_{20} \end{rcases} \frac{AB^2}{AC^2} = \frac{20 \times 16}{20 \times 4} = 4 \Rightarrow \frac{AB}{AC} = 2$

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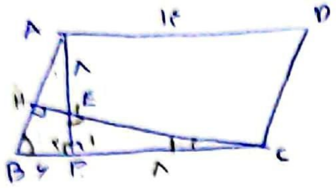


$\boxed{AF = ? \Rightarrow AF = EF + AE = 4 + 8 = 12}$

$\overset{\square}{ABCD} \Rightarrow AD = BC = 14$

$BC = BF + FC \Rightarrow FC = 8$

$\overset{\triangle}{BHC} \Rightarrow \hat{C}_1 + \hat{B} = 90$

$\overset{\triangle}{EFC} \Rightarrow \hat{E}_1 + \hat{C}_1 = 90$ } $\hat{B} = \hat{E}_1$

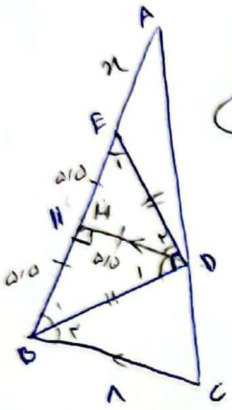
$\hat{F}_1 = \hat{F}_2 = 90$

$\Rightarrow \overset{\triangle}{ABF} \sim \overset{\triangle}{CFE} \Rightarrow \frac{EF}{6} = \frac{8}{8+EF} \Rightarrow (EF)^2 + 8EF = 48$, $EF = x$

$x^2 + 8x - 48 = 0$

$(x+12)(x-4) = 0$: $x = -12$ غ ق ق ، $x = 4$ ق ق

۶



AE? = 6/6

BD نیمساز $\Rightarrow \hat{B}_1 = \hat{B}_2$

$BH \parallel BC \Rightarrow \hat{D}_1 = \hat{B}_2$ } $\hat{D}_1 = \hat{B}_1$ (I) $\Rightarrow \overset{\triangle}{BHD}$ متساوی الساقین $\Rightarrow BH = DH$

$\overset{\triangle}{BHD} \Rightarrow \hat{B}_1 + \hat{D}_1 = 90° \xRightarrow{(I)} 2\hat{B}_1 = 90° \Rightarrow \hat{B}_1 = \hat{D}_1 = 45°$

$\hat{D}_1 + \hat{D}_2 = 90° \Rightarrow \hat{D}_2 = 45°$

$\overset{\triangle}{BDE} \Rightarrow \hat{E}_1 + \hat{B} = 90 \Rightarrow \hat{E}_1 = 45°$ } $\overset{\triangle}{BDE}$ مثلث متساوی الساقین $\Rightarrow$ DH میانه‌ی وارد بر وتر نیز است $\Rightarrow DH = BH = HE = \frac{11}{2} = 5/5$

$HD \parallel BC \Rightarrow \frac{x + 5/5}{5/5 + 11 + x} = \frac{5/5}{8} \xRightarrow{\text{تفضیل صورت در مخرج}} \frac{x+5/5}{5/5} = \frac{5/5}{2/5} \xRightarrow{\text{تفضیل مخرج در صورت}} \frac{x}{5/5} = \frac{3}{2/5} \Rightarrow x = \frac{33}{5} = 6/6$

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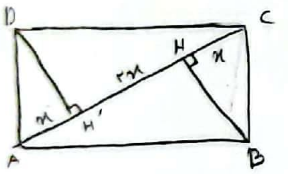

$S_{\overset{\triangle}{ABC}} = \frac{BC \times AB}{2}$

$S_{\overset{\triangle}{ADC}} = \frac{AD \times DC}{2}$

$\overset{\square}{ABCD} \Rightarrow BC = AD , DC = AB$

$\Rightarrow S_{\overset{\triangle}{ADC}} = S_{\overset{\triangle}{ABC}}$

$S_{\overset{\square}{ABCD}} = S_{\overset{\triangle}{ADC}} + S_{\overset{\triangle}{ABC}}$

$\Rightarrow S_{\overset{\square}{ABCD}} = 2S_{\overset{\triangle}{ABC}}$ (I)

$S_{\overset{\triangle}{ABC}} = \frac{BH \times AC}{2}$

$S_{\overset{\triangle}{BHC}} = \frac{BH \times CH}{2}$ } $\frac{S_{\overset{\triangle}{ABC}}}{S_{\overset{\triangle}{BHC}}} = \frac{AC}{CH} = \frac{x+x+2x}{x} = 4 \Rightarrow \frac{S_{\overset{\square}{ABCD}}}{S_{\overset{\triangle}{BHC}}} \overset{(I)}{=} \frac{2S_{\overset{\triangle}{ABC}}}{S_{\overset{\triangle}{BHC}}} = 8$

(کوچکترین مثلث ایجاد شده)

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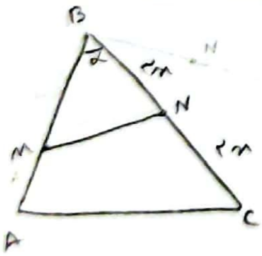


$2BN = 3NC$

$\frac{BM}{AM} ? = \frac{5}{4}$

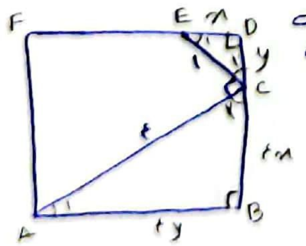
$S_{\overset{\triangle}{ABC}} = 3S_{BMN} \rightarrow \cancel{\tfrac{1}{2}} BC \times AB \cancel{\sin\alpha} = 3 \times \cancel{\tfrac{1}{2}} \times BM \times BN \cancel{\sin\alpha}$

$\underbrace{BC}_{5x} \times AB = 3BM \times \underbrace{BN}_{3x} \Rightarrow 5AB = 9BM$

$\frac{BM}{AB} = \frac{5}{9}$

$\frac{BM}{AM + MB} = \frac{5}{9} \xRightarrow{\text{تفضیل صورت در مخرج}} \frac{BM}{AM} = \frac{5}{4}$

۹



مساحت مربع ؟

$\hat{C}_1 + \hat{C}_2 = 90°$

$\hat{C}_1 + \hat{E}_1 = 90°$ } $\hat{E}_1 = \hat{C}_2$

$\overset{\square}{AFDB} \Rightarrow \hat{B} = \hat{D} = 90°$

$\Rightarrow \overset{\triangle}{EDC} \sim \overset{\triangle}{CBA} \Rightarrow \frac{AC}{EC} = \frac{CB}{ED} = \frac{BA}{DC} = \frac{4}{1}$

$ED = x , DC = y$

$BC = 4x , AB = 4y$

$\overset{\triangle}{CDE} \Rightarrow x^2 + y^2 = 1$

$\overset{\square}{AFDB} \Rightarrow AB = BD \Rightarrow 4y = 4x + y \Rightarrow y = \frac{4}{3}x$ } $x^2 + \frac{16}{9}x^2 = 1 \rightarrow \frac{25}{9}x^2 = 1$

$y = \frac{4}{5} , x = \frac{3}{5}$

$S_{\text{مربع}} = (4y)^2 = 16y^2 = 16 \times \frac{16}{25} = 10/24$

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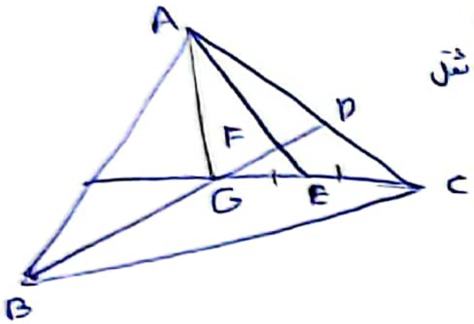

$GE = EC$

G مرکز ثقل

$\frac{BD}{FD} = ?$

$\overset{\triangle}{AGC} \Rightarrow GE = EC \Rightarrow AE$ میانه‌ی مثلث (I)

G مرکز ثقل $\Rightarrow$ G محل برخورد میانه‌ها است $\Rightarrow \overset{\triangle}{ABC} \Rightarrow BD$ میانه وارد بر AC $\Rightarrow AD = DC \Rightarrow GD$ در مثلث $\overset{\triangle}{AGC}$ میانه است (II)

$(I), (II) \Rightarrow$ F مرکز ثقل مثلث $\overset{\triangle}{AGC}$ است $\Rightarrow \frac{GF}{FD} = \frac{2}{1} \xrightarrow[\text{مخرج در صورت}]{\text{ترکیب}} \frac{\overbrace{GF+FD}^{GD}}{FD} = \frac{3}{1}$ (1)

G مرکز ثقل مثلث $\overset{\triangle}{ABC}$ است $\Rightarrow \frac{BG}{GD} = \frac{2}{1} \xRightarrow{\text{ترکیب مخرج در صورت}} \frac{\overbrace{BG+GD}^{BD}}{GD} = \frac{3}{1}$ (2)

$(1), (2) \Rightarrow \frac{\cancel{GD}}{FD} \times \frac{BD}{\cancel{GD}} = 3 \times 3 \Rightarrow 9 = \frac{BD}{FD}$