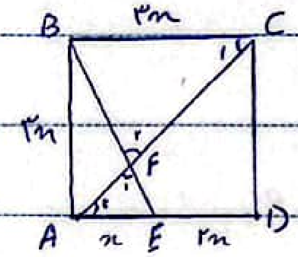
آروشا پاسبان یازدهم دفتر A تکلیف شماره ۲۱

۱ـ



$AE = x \to ED = 2x \to AD = BC = 3x$

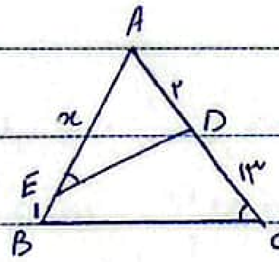
$\left.\begin{aligned}\hat{F}_1 &= \hat{F}_2\\ \hat{A}_1 &= \hat{C}_1\end{aligned}\right\} \Rightarrow \overset{\Delta}{AFE} \sim \overset{\Delta}{FBC}$ نسبت تشابه $= \frac{3x}{x} = 3$

$BE = \sqrt{(3x)^2 + (x)^2} = \sqrt{10}\,x \to EF = \frac{1}{4} EB = \frac{\sqrt{10}}{4} x$

$AC = \sqrt{(3x)^2 + (3x)^2} = 3\sqrt{2}\,x \to AF = \frac{1}{4} AC = \frac{3\sqrt{2}}{4} x$

$\frac{EF}{AF} = \frac{\frac{\sqrt{10}}{4} x}{\frac{3\sqrt{2}}{4} x} = \frac{\sqrt{5}}{3}$

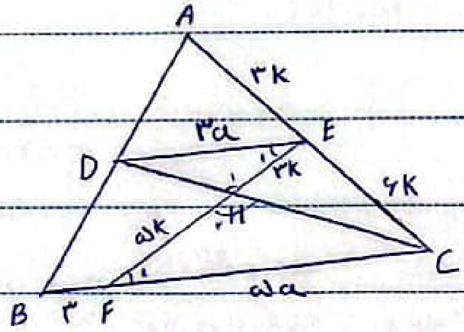
۲ـ



$\left.\begin{aligned}\hat{A} &= \hat{A} \text{ مشترک}\\ \hat{E} &= \hat{C}\end{aligned}\right\} \Rightarrow \overset{\Delta}{ADE} \sim \overset{\Delta}{ABC} \to \frac{2}{x+1} = \frac{x}{15}$

$\to 30 = x^2 + x = x(x+1) \to x = 5$

۳ـ



$3y = 5x \to x = 3k,\ y = 5k$

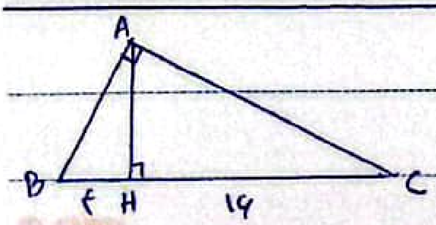
$\left.\begin{aligned}\hat{H}_1 &= \hat{H}_2\\ \hat{E}_1 &= \hat{F}_1\end{aligned}\right\} \Rightarrow \overset{\Delta}{EDH} \sim \overset{\Delta}{FHC} \to \frac{3k}{5k} = \frac{3}{5} =$ نسبت تشابه

$\frac{3a}{3+5a} = \frac{3k}{9k} = \frac{1}{3}$

$9a = 3 + 5a \to 4a = 3 \to a = \frac{3}{4}$

$BC = 3 + 5\left(\frac{3}{4}\right) = 3 + \frac{15}{4} = \frac{27}{4} = 6{,}75$

۴ـ

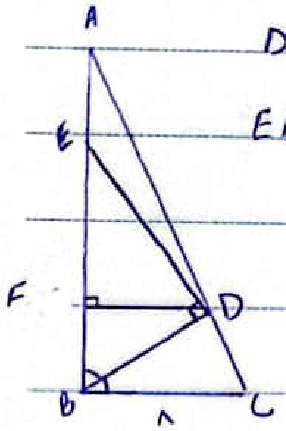


$AC^2 = HC \times BC = 16 \times 20$

$AB^2 = BH \times BC = 4 \times 20 \to \frac{AC^2}{AB^2} = 4 \to \frac{AC}{AB} = 2$

$$\left.\begin{matrix}\hat{E}_1 = \hat{E}_2 \\ \hat{F} = ۹۰^\circ\end{matrix}\right\} \rightarrow B\hat{A}F \sim E\hat{C}F \rightarrow \frac{BF}{EF} = \frac{AF}{CF} \rightarrow \frac{۶}{x} = \frac{۸+x}{۸} \quad (EF = x)$$ ـ۵

$$\rightarrow x(x+۸) = ۴۸ \rightarrow x = ۴ \rightarrow AF = ۸ + x = ۸ + ۴ = ۱۲$$

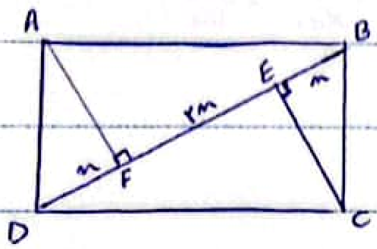


$$DF \parallel BC \rightarrow \hat{B} = \hat{F} = ۹۰ \Rightarrow \frac{۹۰}{۲} = ۴۵ = D_1 = D_2 = B_1 = B_2$$ ـ۶

$$ED = x = DB \rightarrow \hat{E} = \hat{B}_1 = ۴۵ \Rightarrow \left.\begin{matrix}EF = FD \\ BF = FD\end{matrix}\right\} \Rightarrow EF = BF = \frac{۱۱}{۲} = ۵/۵$$

$$DF \parallel BC \longrightarrow \frac{\overset{x+۵/۵}{AF}}{\underset{x+۱۱}{AB}} = \frac{\overset{۵/۵}{FD}}{\underset{۸}{BC}} \rightarrow ۸x + ۴۴ = ۵/۵x + ۶۰/۵$$

$$\rightarrow x = \frac{۱۶/۵}{۲/۵} = ۶/۶ \rightarrow AF = ۶/۶$$



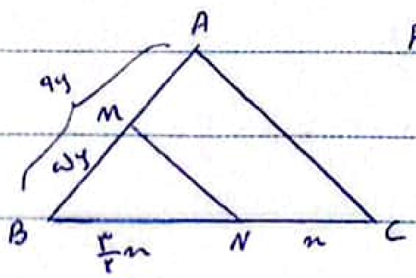
$$AF = CE = h$$ ـ۷

$$A\hat{D}F = B\hat{C}E = \frac{h \times x}{۲} \qquad A\hat{B}F = E\hat{C}D = \frac{h \times ۳x}{۲}$$

$$S_{ABCD} = ۲\left(\frac{h \times x}{۲}\right) + ۲\left(\frac{h \times ۳x}{۲}\right) = ۴hx$$

$$S_{ADF} = \frac{h \times x}{۲}$$

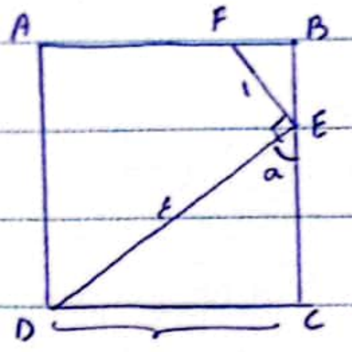
$$\frac{S_{ABCD}}{S_{ADF}} = \frac{۴hx}{\frac{h \times x}{۲}} = ۸$$



$$BC = \frac{۳}{۲}x + x = \frac{۵}{۲}x \qquad \frac{S_{ABC}}{S_{MBN}} = \frac{\cancel{\sin B} \times AB \times \overset{\frac{۵}{۲}x}{\cancel{BC}}}{\cancel{\sin B} \times BM \times \underset{\frac{۳}{۲}x}{\cancel{BN}}} = ۳$$ ـ۸

$$\rightarrow \frac{AB}{BM} = ۳ \times \frac{۳}{۵} = \frac{۹}{۵} \rightarrow AB = ۹y, \; BM = ۵y$$

$$AB - BM = ۴y = AM \qquad \frac{BM}{AM} = \frac{۵y}{۴y} = \frac{۵}{۴}$$



۹-

$\hat{B} = \hat{C} = ۹۰^\circ$

$\left.\begin{array}{l}\hat{a} + \hat{B} = ۹۰ \\ \hat{B} + \hat{E}_1 = ۹۰\end{array}\right\} \Rightarrow \hat{E}_2 = \hat{a}$

$\left.\begin{array}{l}E_2 + D_1 = ۹۰ \\ \hat{a} + B = ۹۰\end{array}\right\} \Rightarrow \hat{D}_2 = \hat{B}$

$\Rightarrow \overset{\triangle}{FBE} \sim \overset{\triangle}{ECD}$

$\to \frac{۴}{۱} = \frac{DE}{BE} = \frac{EC}{FB}$ $\quad \sqrt{۱۶ - a^۲} = EC \quad EB = a - \sqrt{۱۶ - a^۲}$

$a = ۴a - ۴\sqrt{۱۶ - a^۲} \to ۳a = ۴\sqrt{۱۶ - a^۲}$

$۹a^۲ = ۱۶ \times ۱۶ - ۱۶a^۲ \to ۲۵a^۲ = ۱۶^۲ \to a^۲ = \frac{۲۵۶}{۲۵} = ۱۰/۲۴$

۱۰-

میانه $BD \longrightarrow AD = DC$ $\quad$ ① $\to GD \to$ میانه

میانه $AH \longrightarrow$ از G میگذرد $\quad$ ② $\to GE = EC$ فرض $\to AE$ میانه

①,② $\to \overset{\triangle}{AGC}$ $\left\{\begin{array}{l} GD \text{ میانه} \\ AE \text{ میانه}\end{array}\right.$ $\Rightarrow F$ نقطه ثقل $\longrightarrow$ $FD = n$, $GF = ۲n$ $\quad AC$ مرکز ثقل $\to BG = ۶n$

$\frac{BD}{FD} = \frac{۶n + ۲n + n}{n} = ۹$