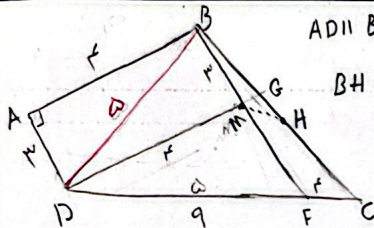


$$DE \parallel BC \rightarrow \frac{DE}{BC} = \frac{AD}{AB} = \frac{4}{12}$$

BD ← ارتفاع مثلث BDE و AB ارتفاع مثلث BCE

$$\Rightarrow \frac{S_{BCE}}{S_{BDE}} = \frac{AB}{AD} = \frac{12}{4} = 3$$

۱



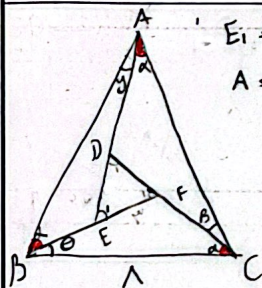
AD ∥ BF, AB ∥ DG

BH = HC

دو مثلث DFM و DBM هم‌بند هستند براساس یک ضلع و زاویه مشترک برابر
دو مثلث BMH و BFC متشابه هستند. ← زاویه B مشترک و

$$\frac{BH}{BC} = \frac{BM}{BF} = \frac{1}{2} \quad \text{و} \quad \frac{MH}{FC} = \frac{1}{2} \quad \Rightarrow \quad \frac{MH}{FC} = \frac{1}{2} \quad \Rightarrow \quad \frac{MH}{FC} = \frac{1}{2}$$

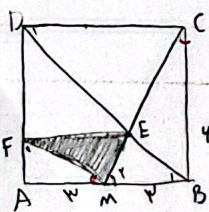
۲



$$\left. \begin{array}{l} \angle E = \angle A + \alpha / \angle D = \alpha + \beta / \angle F = \alpha + \theta \\ \angle A = \angle A + \alpha / \angle C = \alpha + \beta / \angle B = \alpha + \theta \end{array} \right\} \Rightarrow ABC \sim DFE$$

$$\frac{DE}{BC} = \frac{EF}{AB} = \frac{DF}{AC} \Rightarrow \frac{3}{AB} = \frac{4}{12} \Rightarrow AB = \frac{3 \times 12}{4} = 9$$

۳



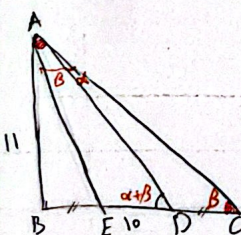
$$\left. \begin{array}{l} \angle C = \angle A \\ \angle A = \angle C \end{array} \right\} \Rightarrow \triangle ABC \sim \triangle AEF \Rightarrow \angle A = \angle C$$

$$\frac{AF}{AB} = \frac{AE}{CB} \Rightarrow AF = \frac{9}{4} \times \frac{3}{4} = \frac{27}{16} / MF^2 = AF^2 + AM^2 \Rightarrow \frac{9}{4} + 9 = MF^2, MF = \frac{3\sqrt{5}}{2}$$

$$MC^2 = BC^2 + MB^2 = 9 + 36 \Rightarrow MC = 3\sqrt{5}$$

$$S = \frac{ME \times MF}{2} = \frac{\frac{3\sqrt{5}}{2} \times \frac{3\sqrt{5}}{2}}{2} = \frac{45}{8}$$

۴



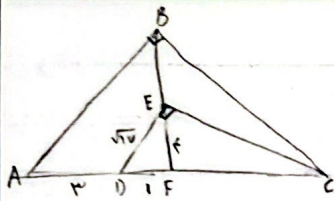
$$\left. \begin{array}{l} \angle C = \angle A \\ \angle E = \angle E \end{array} \right\} \Rightarrow \triangle AED \sim \triangle AEC$$

$$\frac{ED}{AE} = \frac{AD}{AC} = \frac{AE}{EC} \Rightarrow AE^2 = 10(10 + DC)$$

$$AE^2 = AB^2 + BE^2 \Rightarrow AE^2 = 10 + BE^2 \Rightarrow 10 + BE^2 = 10(10 + DC) \Rightarrow 10 + DC^2 = 10 + 10DC$$

$$DC^2 - 10DC + 10 = 0 \Rightarrow (DC - 3)(DC - 7) = 0 \Rightarrow DC = 3 \text{ یا } 7$$

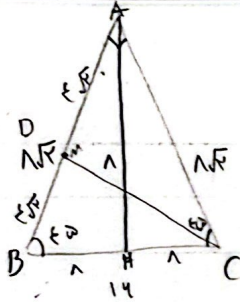
۵



$$ED^2 = DF \times DC \rightarrow (\sqrt{5})^2 = 1 \times DC \Rightarrow DC = 5, FC = 14$$

$$BC^2 = FC \times AC \rightarrow BC^2 = 14 \times 20 = 280 \Rightarrow BC = \sqrt{280} = 14\sqrt{2}$$

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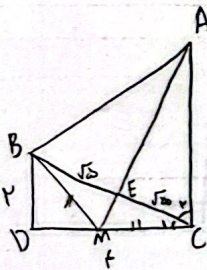


$$AB^2 = BH^2 + AH^2 = 14^2 + 4^2 \Rightarrow AB = 15 \Rightarrow AC = 15\sqrt{2}$$

میان وارد بر قاعده در مثل متساوی الساقین ارتفاع مثلث هم هست. در نتیجه ضلع BC را داشت می‌تواند و دو مثلث قائم الزامی هم تشکیل می‌دهد.

$$A = 90^\circ \Rightarrow AC^2 + AD^2 = DC^2 \rightarrow (15\sqrt{2})^2 + (5\sqrt{2})^2 = 140$$

$$\Rightarrow DC = \sqrt{140} = 10\sqrt{2}$$

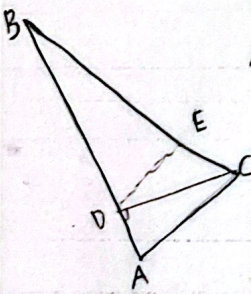


$$BC^2 = CD^2 + BD^2 = 14^2 + 4^2 = 200 \Rightarrow BC = 10\sqrt{2}$$

ME ارتفاع مثلث BMC است و چون مثلث متساوی الساقین است میان مثلث هم می‌باشد و در نتیجه $BE = EC = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

$$C_1 + C_2 = 90^\circ \quad \cot C_1 = 2 \Rightarrow \tan C_2 = 2$$

$$\tan C_2 = \frac{AE}{EC} = 2 \Rightarrow AE = 2\sqrt{2} \quad , \quad S_{ABE} = AE \times EC = 2\sqrt{2} \times \sqrt{2} \times \frac{1}{2} = 2$$

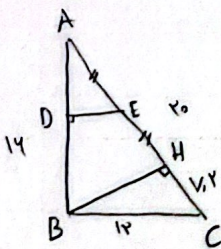


$$AB^2 = AC^2 + BC^2 \rightarrow 9 + 11 = 20 \quad , \quad AB = \sqrt{20} \Rightarrow AB = 2\sqrt{5}$$

$$AC^2 = AD \times AB \Rightarrow 9 = AD \times 2\sqrt{5} \Rightarrow AD = \frac{9\sqrt{5}}{2} \Rightarrow BD = 2\sqrt{5} - \frac{9\sqrt{5}}{2} = \frac{4\sqrt{5}}{2} = 2\sqrt{5}$$

$$BD^2 = BE \times BC \rightarrow \left(\frac{4\sqrt{5}}{2}\right)^2 = BE \times 9 \Rightarrow BE = 1$$

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$$AB^2 + BC^2 = AC^2 \rightarrow 14^2 + 12^2 = 20^2 \quad , \quad AC = 20$$

$$BC^2 = CH \times AC \rightarrow 144 = CH \times 20 \Rightarrow CH = \frac{36}{5} \Rightarrow AH = 12 - \frac{36}{5} = \frac{24}{5}$$

$$AE = EH = \frac{AH}{2} = \frac{12}{5} \quad , \quad \frac{AE}{AC} = \frac{DE}{BC} \Rightarrow DE = \frac{\frac{12}{5} \times 12}{20} = \frac{36}{25}$$

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