

۱۴

$DE \parallel BC \rightarrow \triangle ADE \sim \triangle ABC \rightarrow \frac{S_{ADE}}{S_{ABC}} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{5}{14}\right)^2 = \frac{25}{196}$ (1)

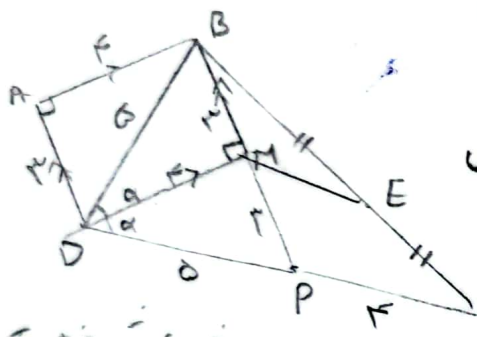
$\rightarrow S_{ABC} = \frac{196}{25} S_{ADE} \Rightarrow$

$\frac{S_{ADE}}{S_{BDE}} = \frac{\frac{1}{2} EH \times AD}{\frac{1}{2} EH \times BD} = \frac{AD}{BD} \rightarrow \frac{x}{S_{BDE}} = \frac{5}{9} \rightarrow S_{BDE} = \frac{9}{5} x$

$S_{BEC} = S_{ABC} - S_{ADE} - S_{BDE} = \frac{196}{25} x - x - \frac{9}{5} x = \frac{146x - 25x - 45x}{25} = \frac{116x}{25}$

$S_{BCF} = \frac{14}{25} x$

$\frac{S_{BCE}}{S_{BDE}} = \frac{\frac{116}{25} x}{\frac{9}{5} x} = \frac{116}{9} = \frac{12}{9} = 1.3$



ABMD مستطیل است $DM \parallel AB$ و $BM \parallel AD \rightarrow$ (۲)

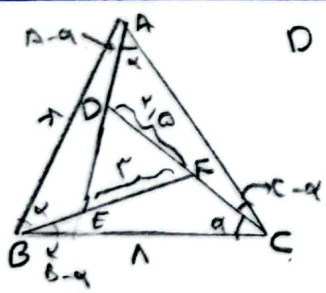
$AD=3, AB=4 \rightarrow DB = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$\angle BDC = 2\angle BDM \rightarrow \angle BDM = \angle DMP = \alpha \rightarrow$

DM نیماز و ارتفاع مثلث BDP است پس مثلث BDP در رأس D متساوی الساقین است پس DM جاده نیز است

درایم $BM = MP = 3$ در مثلث BPC چون M و E وسط اضلاع مثلث هستند

$ME = \frac{1}{2} PC \rightarrow ME = \frac{1}{2} \times 4 = 2$ پس طبق معکوس تالس داریم $ME \parallel PC$ و داریم



$DF = 2/5, EF = 3, \angle AEF = \angle CAE = \angle BCD, AB = ?$ (۳)

$\angle BCF: \angle EFD = \angle EFD = \angle B - \alpha + \alpha = \angle B$

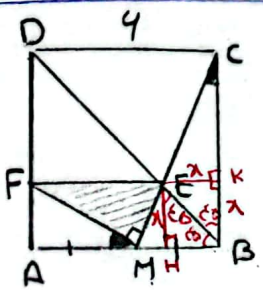
$\angle AEB: \angle DEF = \angle DEF = \angle A - \alpha + \alpha = \angle A$

$\angle ACD: \angle EDF = \angle EDF = \angle C - \alpha + \alpha = \angle C$

بنابراین مثلث DEF متساوی الساقین است و ABC و DEF متساوی الساقین

$\frac{DF}{BC} = \frac{EF}{AB} \rightarrow \frac{2/5}{x} = \frac{3}{x} \rightarrow$

$x = \frac{3 \times 5}{2/5} = 9.4$



$\triangle BCM \sim \triangle AFM$

$\angle BCM = \angle AMF, \angle B = \angle A = 90^\circ \rightarrow \triangle BCM \sim \triangle AFM \rightarrow \frac{BM}{AF} = \frac{BC}{AM} \rightarrow \frac{5}{AF} = \frac{4}{4} \rightarrow$

$AF = \frac{5}{4}$

$MF^2 = AM^2 + AF^2 \Rightarrow MF^2 = 3^2 + \left(\frac{5}{4}\right)^2 = 9 + \frac{25}{16} = \frac{144 + 25}{16} = \frac{169}{16}$

$MF = \frac{13}{4}$

از E بر B و BM ارتفاع رسم می کنیم. در مربع قطر منبسط است پس با توجه به شکل می توانیم

$$EK \parallel MB \rightarrow \frac{EK}{BC} = \frac{EM}{BM} \rightarrow$$

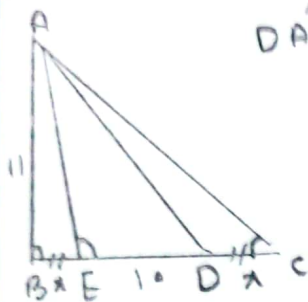
در مثل $EKBH$ بر من به ضلع x است و داریم

$$\frac{9-x}{9} = \frac{x}{x} \rightarrow 9-x = x \rightarrow 9 = 2x \rightarrow x = \frac{9}{2} \Rightarrow HH = \frac{9}{2} - x = \frac{9}{2} - \frac{9}{2} = 0$$

$$ME^2 = EH^2 + HH^2 = \left(\frac{9}{2}\right)^2 + 0 = \frac{81}{4} \rightarrow ME = \frac{9}{2}$$

در مثل EMH داریم

$$S_{EFM} = \frac{1}{2} \cdot MF \cdot ME = \frac{1}{2} \cdot \frac{9}{2} \cdot \frac{9}{2} = \frac{81}{8} = 10.125$$



$$\widehat{DAE} = \widehat{ACD}, BE = DC, DC = x$$

(5)

$$\left. \begin{array}{l} \widehat{DAE} = \widehat{ACD} \\ \widehat{E_1} = \widehat{E} \end{array} \right\} \rightarrow \triangle ADE \sim \triangle AEC \rightarrow \frac{AD}{AC} = \frac{AE}{CE} = \frac{ED}{AE}$$

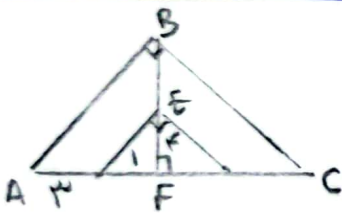
$$\rightarrow BE = DC = x$$

(5)

$$\frac{AE}{CE} = \frac{ED}{AE} \rightarrow AE^2 = ED \cdot CE = 10 \cdot (10 + x) = 100 + 10x$$

$$\triangle ABE \rightarrow AE^2 = AB^2 + BE^2 \rightarrow AE^2 = 11^2 + x^2 \xrightarrow{(1)} 121 + x^2 = 100 + 10x$$

$$x^2 - 10x + 21 = 0 \rightarrow \begin{cases} x = 3 \\ x = 7 \end{cases} \checkmark \quad DC = 7$$



$$DF = 1, EF = 6, AD = 3, \angle B = \angle C = 90^\circ$$

(4)

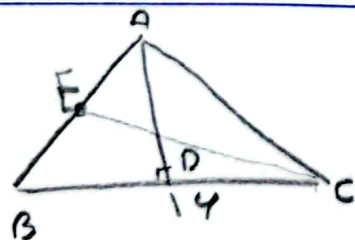
در مثل قائم الزاویه DEC ، ارتفاع وارد بر وتر است

$$EF^2 = DF \cdot FC \rightarrow 6^2 = 1 \cdot FC \rightarrow FC = 36$$

و داریم:

از طرفی در مثل قائم الزاویه ABC ، ارتفاع وارد بر وتر است پس

$$BC^2 = CF \cdot CA = 36 \cdot (36 + 6) = 36 \cdot 42 \rightarrow BC = 6\sqrt{42} = 6 \cdot \sqrt{6 \cdot 7} = 6\sqrt{6} \cdot \sqrt{7} = 6\sqrt{42}$$



$$AD = \frac{1}{4} BC = \frac{1}{4} \cdot 14 = 3.5$$

$$BD = DC = \frac{BC}{2} = 3.5$$

و $AD \perp BC$ عمود است

در مثل متساوی الساقین ABC که نه وارد بر وتر است، ارتفاع هم میانه است

$$\triangle DBA : AB^2 = AD^2 + BD^2 = 3.5^2 + 3.5^2 = 12.25 \rightarrow AB = 3.5\sqrt{2}$$

$$a = 14, AC = \sqrt{2} \cdot 3.5 = 4.95, AB = c = 3.5\sqrt{2}$$

CE میانه نظیر AB

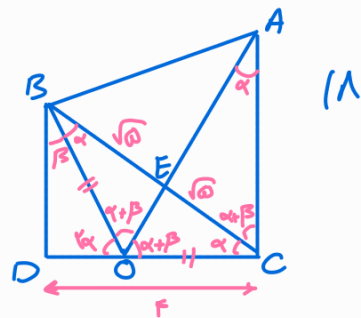
$$m_c = \sqrt{\frac{2a^2 + 2b^2 - c^2}{4}} = \sqrt{\frac{2(14)^2 + 2(4.95)^2 - (3.5\sqrt{2})^2}{4}} = \sqrt{\frac{440}{4}} = \sqrt{110} = 10.488$$

$$\alpha + \beta = 90^\circ$$

$$BD^2 + DC^2 = BC^2 \rightarrow BC = r\sqrt{2} = rBE = rEC \\ \rightarrow BE = EC = \sqrt{2}r$$

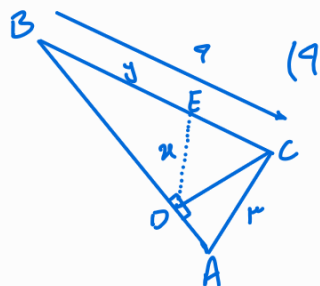
$$\frac{r}{\sqrt{2}} = \frac{r}{AE} = \frac{r\sqrt{2}}{2} \rightarrow AE = r\sqrt{2}$$

$$S_{ABE} = \frac{1}{2} AE \times BE = 2$$



$$\frac{\alpha}{r} = \frac{y}{r} \rightarrow y = r\alpha$$

$$r^2 = y(9-y) = r\alpha(9-r\alpha) \rightarrow \alpha = r, y = 1,1$$



$$AB^2 + BC^2 = AC^2 \rightarrow AC = r_0$$

$$\triangle ADE \sim \triangle ABH \rightarrow \begin{cases} \hat{A} = \hat{A} \\ \hat{D} = \hat{H} = 90^\circ \end{cases} \rightarrow \frac{DE}{BH} = \frac{AE}{14} \quad (1)$$

$$S_{ABC} = \frac{AB \times BC}{2} = \frac{BH \times AC}{2} \rightarrow BH = 9,4$$

$$\triangle ABC \rightarrow 14^2 = AH \times r_0 \rightarrow AH = 12,1 \rightarrow AE = \frac{AH}{r} = 4,6$$

$$(1) \rightarrow \frac{DE}{BH} = \frac{AE}{14} = \frac{DE}{9,4} = \frac{4,6}{14} \rightarrow DE = 5,8$$

