

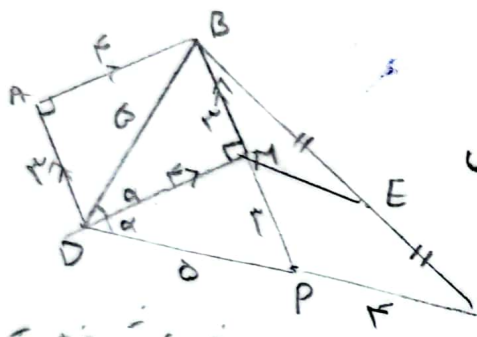
$DE \parallel BC \rightarrow \triangle ADE \sim \triangle ABC \rightarrow \frac{S_{ADE}}{S_{ABC}} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{5}{14}\right)^2 = \frac{25}{196}$
 $\rightarrow S_{ABC} = \frac{196}{25} S_{ADE} \Rightarrow$

$\frac{S_{ADE}}{S_{BDE}} = \frac{\frac{1}{2} EH \times AD}{\frac{1}{2} EH \times y} = \frac{5}{y} \rightarrow \frac{x}{y} = \frac{5}{y} \rightarrow S_{BDE} = \frac{y}{5} x$

$S_{BEC} = S_{ABC} - S_{ADE} - S_{BDE} = \frac{196}{25} x - x - \frac{y}{5} x = \frac{196x - 25x - 5yx}{25} = \frac{146x}{25}$

$S_{BCF} = \frac{14}{5} x$

$\frac{S_{BCE}}{S_{BDE}} = \frac{\frac{146}{25} x}{\frac{y}{5} x} = \frac{146}{5y} = \frac{12}{5} = 2,4$



$DM \parallel AB$ و $BM \parallel AD \rightarrow$ مستطیل است $ABMD$ (۲)

$AD=3, AB=4 \rightarrow DB = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

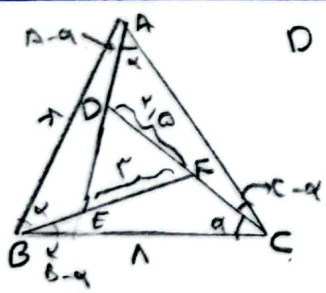
$\angle BDC = 2\angle BDM \rightarrow \angle BDM = \angle DMP = \alpha$

DM نیمساز ارتفاع مثلث BDP است پس مثلث BDP

در رأس D متساوی الساقین است پس DM عمود نیز هست

داریم $BM = MP = 4$ در مثل BCP چون M و E وسط اضلاع مثلث هستند

پس طبق میانه ها $ME \parallel PC$ داریم $ME = \frac{1}{2} PC \rightarrow ME = \frac{1}{2} \times 4 = 2$



$DF = 2/5, EF = 3, \angle AFB = \angle CAE = \angle BCD, AB = ?$ (۳)

$\triangle BCF: \angle EFD = \angle EFD = \angle B - \alpha + \alpha = \angle B$

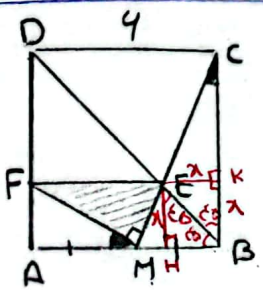
$\triangle AEB: \angle DEF = \angle DEF = \angle A - \alpha + \alpha = \angle A$

$\triangle ACD: \angle EDF = \angle EDF = \angle C - \alpha + \alpha = \angle C$

بنابرین مثلث DEF متساوی الساقین است ABC و DEF

$\frac{DF}{BC} = \frac{EF}{AB} \rightarrow \frac{2/5}{4} = \frac{3}{x} \rightarrow$

$x = \frac{3 \times 4}{2/5} = 30$



$\triangle BCM \sim \triangle AFM$

$\angle BCM = \angle AFM, \angle B = \angle A = 90^\circ \rightarrow \triangle BCM \sim \triangle AFM \rightarrow \frac{BM}{AF} = \frac{BC}{AM} \rightarrow \frac{4}{AF} = \frac{4}{4} \rightarrow$

$AF = 4$

$MF^2 = AM^2 + AF^2 \Rightarrow MF^2 = 2^2 + (4)^2 = 4 + 16 = 20 = \frac{40}{2} = \frac{4 \times 5}{2}$

$MF = \frac{2}{\sqrt{5}}$

از E به B و BM ارتفاع رسم می کنیم. در مربع قطر منبسط است پس با توجه به شکل می توانیم

$$EK \parallel MB \rightarrow \frac{EK}{BC} = \frac{EM}{BM} \rightarrow$$

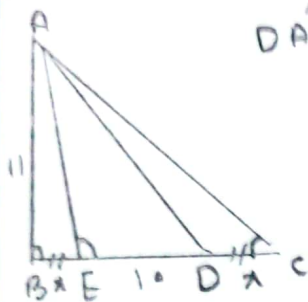
در مثل $EKBH$ بر من به ضلع x است و داریم

$$\frac{9-x}{9} = \frac{x}{x} \rightarrow 9-x = x \rightarrow 9 = 2x \rightarrow x = \frac{9}{2} \Rightarrow HH = \frac{9}{2} - x = \frac{9}{2} - \frac{9}{2} = 0$$

$$ME^2 = EH^2 + MH^2 = \left(\frac{9}{2}\right)^2 + 1^2 = \frac{85}{4} \rightarrow ME = \sqrt{\frac{85}{4}}$$

در مثل EMH داریم

$$S_{EFM} = \frac{1}{2} HF \cdot ME = \frac{1}{2} \times \frac{9}{2} \times \sqrt{\frac{85}{4}} = \frac{15}{4} \sqrt{85}$$



$$\widehat{DAE} = \widehat{ACD}, BE = DC \quad DC = x$$

(5)

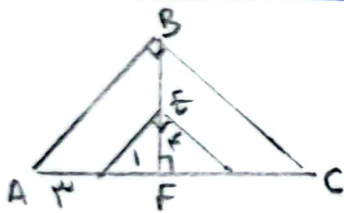
$$\left. \begin{array}{l} \widehat{DAE} = \widehat{ACD} \\ \widehat{E_1} = \widehat{E} \end{array} \right\} \rightarrow \triangle ADE \sim \triangle AEC \rightarrow \frac{AD}{AC} = \frac{AE}{CE} = \frac{ED}{AE}$$

$$\rightarrow BE = DC = x$$

$$\frac{AE}{CE} = \frac{ED}{AE} \rightarrow AE^2 = ED \cdot CE = 1 \cdot (1+x) = 1 + x \quad (1)$$

$$\triangle ABE \rightarrow AE^2 = AB^2 + BE^2 \rightarrow AE^2 = 11^2 + x^2 \xrightarrow{(1)} 1 + x = 121 + x^2$$

$$x^2 - 10x + 120 = 0 \rightarrow \begin{cases} x = 2 \\ x = 7 \end{cases} \rightarrow DC = 7$$



$$DF = 1, EF = 6, AD = 3 \quad \angle ABC = \angle CED = 90^\circ$$

(4)

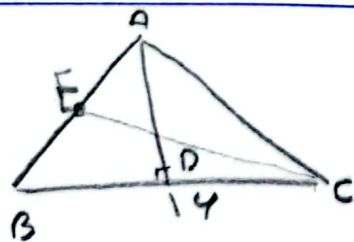
در مثل قائم الزاویه DEC ، ارتفاع وارد بر وتر است

$$EF^2 = DF \cdot FC \rightarrow 6^2 = 1 \cdot FC \rightarrow FC = 36$$

و داریم:

از طرفی در مثل قائم الزاویه ABC ، ارتفاع وارد بر وتر است پس

$$BC^2 = CF \cdot CA = 36 \cdot (36 + 6) = 36 \cdot 42 \rightarrow BC = 6\sqrt{42} = 6 \times 2\sqrt{10.5} = 12\sqrt{10.5}$$



$$AD = \frac{1}{4} BC = \frac{1}{4} \times 14 = 3.5$$

$$BD = DC = \frac{BC}{2} = 7$$

و $AD \perp BC$ عمود است

در مثل متساوی الساقین ABC که نه وارد بر وتر است، ارتفاع هم می افتد

$$\triangle DBA : AB^2 = AD^2 + BD^2 = 3.5^2 + 7^2 = 12.25 + 49 = 61.25 \rightarrow AB = \sqrt{61.25}$$

$$a = 14, AC = \sqrt{2} \cdot 7 = 7\sqrt{2}, AB = c = 7\sqrt{2}$$

CE میانه نظیر AB

$$m_c = \sqrt{\frac{2a^2 + 2b^2 - c^2}{4}} = \sqrt{\frac{2(14)^2 + 2(7\sqrt{2})^2 - (7\sqrt{2})^2}{4}} = \sqrt{\frac{448}{4}} = \sqrt{112} = 4\sqrt{7}$$