نام و نام خانوادگی ....الناهیت امیری.... پاسخنامه تشریحی تکلیف شماره ..۲۳.. کلاس ..یازدهم ریاضی A..

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$\log_n m = a \qquad \log_{mn} m^2 n = b$

$\log_{mn} m^2 n = \underbrace{\log_{mn} mn}_{1} + \log_{mn} m = 1 + \dfrac{1}{\log_m mn} = 1 + \dfrac{1}{\underbrace{\log_m m}_{1} + \log_m n} = 1 + \dfrac{1}{1 + \dfrac{1}{\underbrace{\log_n m}_{a}}} = 1 + \dfrac{1}{\frac{a+1}{a}} =$

$1 + \dfrac{a}{a+1} \qquad [b] = [1 + \dfrac{a}{a+1}] = 1 + \underbrace{[\dfrac{a}{a+1}]}_{0 < \frac{a}{a+1} < 1} = 1 \Rightarrow \boxed{[b] = 1}$

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الف) $y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}} \rightarrow x > 0$ (I) $\quad \dfrac{x}{\log_{\frac{1}{2}} x} \geq 0 \rightarrow$ $\begin{array}{ccccc} & 0 & & 1 & \\ & \varnothing & + & \varnothing & - \end{array}$ (II) $\quad$ (I)∩(II) $\rightarrow 0 < x < 1 \Rightarrow \boxed{D_f = (0, 1)}$

ب) $y = \dfrac{\log_4 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \rightarrow x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0$ $\begin{array}{ccccc} & -1 & & 2 & \\ + & \varnothing & - & \varnothing & + \end{array}$ $\rightarrow x \in (-\infty, -1) \cup (2, +\infty)$ (I)

$x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \in (-\infty, -1] \cup [1, +\infty)$ (II) $\qquad$ (I)∩(II) $\rightarrow \boxed{x \in (-\infty, -1) \cup (2, +\infty)}$

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$2 \log_x a + \log_a \sqrt{x} = 2 \xrightarrow{x=9} \underbrace{2 \log_9 a}_{\log_3 a} + \log_a 3 = 2 \rightarrow \log_3 a + \dfrac{1}{\log_3 a} = 2 \xrightarrow{(\log_3 a = t)}$

$t + \dfrac{1}{t} = 2 \rightarrow \dfrac{t^2 + 1}{t} = 2 \rightarrow 2t = t^2 + 1 \Rightarrow t^2 - 2t + 1 = 0 \Rightarrow t = 1$

$\Rightarrow \log_3 a = 1 \rightarrow 3^1 = a \Rightarrow \boxed{a = 3}$

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$(\log \frac{5}{4}) x^2 + (\log 9) x - \log 15 = 0$

$|\alpha - \beta| = \dfrac{\sqrt{\Delta}}{|a|} = \dfrac{\sqrt{(\log 9)^2 + 4 \log \frac{5}{4} \log 15}}{\underbrace{|\log \frac{5}{4}|}_{\log 5 - \log 4 = \log 10 - \log 2 - \log 4}} = \dfrac{\sqrt{(2 \overset{0.4}{\log 3})^2 + 4(\overset{1}{\log 10} - \overset{0.3}{\log 2} - \overset{0.6}{\log 4})(\overset{1}{\log 10} - \overset{0.3}{\log 2} + \overset{0.4}{\log 3})}}{\log 10 - \underset{0.3}{\log 2} - \underset{0.6}{\log 4}}$

$= \dfrac{\sqrt{0.64 + 1.32}}{0.3} = \dfrac{\sqrt{1.96}}{0.3} = \dfrac{1.4}{0.3} = \dfrac{14}{3}$

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$\log_{14} 10 = \dfrac{\log_2 10}{\log_2 14} = \dfrac{\overset{\frac{1}{0.5} = 2}{\log_2 5} + \overset{1}{\log_2 2}}{\underset{1}{\log_2 2} + \underset{2.8}{\log_2 7}} = \dfrac{3}{3.8} = \dfrac{15}{19}$

$\left\{ \begin{array}{l} \log_2 7 = 2.8 \\ \log_5 2 = 0.5 \Rightarrow \log_2 5 = \dfrac{1}{\underset{0.5}{\log_5 2}} = 2 \end{array} \right.$

$\log_{3}^{5} = 1.5$

$\log_{2}^{3} = 1.6 \rightarrow \log_{3}^{2} = \frac{1}{\underbrace{\log_{2}^{3}}_{1.6}} = \frac{5}{8}$

$\log_{15}^{6} = \frac{\log_{3}^{6}}{\log_{3}^{15}} = \frac{\overset{1}{\cancel{\log_{3}^{3}}} + \log_{3}^{2}}{\underset{1}{\cancel{\log_{3}^{3}}} + \log_{3}^{5}} = \frac{1+\frac{5}{8}}{1+\frac{3}{2}} = \frac{\frac{13}{8}}{\frac{5}{2}} = \frac{13}{20}$ ۶

$\log_{4}^{12} = \frac{\log_{2}^{12}}{\log_{2}^{4}} = \frac{\overset{2}{\cancel{2\log_{2}^{2}}} + \overbrace{\log_{2}^{3}}^{\frac{3m-1}{2}}}{\underset{2}{\cancel{2\log_{2}^{2}}}} = \boxed{\frac{3m+3}{4}}$ ۷

$\log_{8}^{18} = \log_{8}^{9\times 2} = \log_{\underset{2^3}{8}}^{3^2} + \log_{8}^{2} = m \rightarrow \frac{2}{3}\log_{2}^{3} + \frac{1}{3} = m \Rightarrow \log_{2}^{3} = \frac{3m-1}{2}$

$(0.4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \rightarrow \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5^3}{2^3}\right)^{x^2} \rightarrow \left(\frac{5}{2}\right)^{-2x+1} = \left(\frac{5}{2}\right)^{3x^2} \Rightarrow 3x^2 = -2x+1 \Rightarrow 3x^2+2x-1=0$

$x=-1$

$x=\frac{1}{3}$

$\rightarrow x=-1 \Rightarrow \log_{8}^{(-9+1)} = \log_{8}^{-8}$ غ ق ق ۸

$\rightarrow x=\frac{1}{3} \Rightarrow \log_{8}^{4} = \log_{2^3}^{2^2} = \boxed{\frac{2}{3}}$

$\log_{2}^{3} = a \qquad \log_{8}^{b} = \frac{2}{3}(1+a) = \frac{2}{3}\underbrace{(\log_{2}^{2}+\log_{2}^{3})}_{\log_{2}^{6}} = \log_{2^3}^{6^2} = \log_{8}^{36} \Rightarrow b = 36$

$\log(3b-8) = \log((36\times 3)-8) = \log 100 = 2$ ۹

واسطه حسابی $\rightarrow a = \frac{b+c}{2} \Rightarrow 2a = b+c \rightarrow 2a = (2a\log_{2}^{10}) + c \Rightarrow c = 2a(1-\log_{2}^{10}) = 2a(\log_{2}^{0.2})$

عکس مجموع ریشه‌ها $\rightarrow \frac{1}{x_1+x_2} = \frac{1}{\frac{-b}{-4a}} = \frac{4a}{b} = \log 4 \rightarrow \frac{b}{4a} = \log_{4}^{10} = \frac{1}{2}\log_{2}^{10} \Rightarrow b = 2a\log_{2}^{10}$

$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = \left(\frac{1}{2}\right)^{\frac{1}{8}\left(\frac{c}{a}\right)} = \left(\frac{1}{2}\right)^{\left(\frac{1}{8}\right)\left(\frac{2a\log_{2}^{0.2}}{\cancel{a}}\right)} = \left(\frac{1}{2}\right)^{\left(\frac{1}{4}\log_{2}^{0.2}\right)} = 2^{-\frac{1}{4}\log_{2}^{0.2}} = \left(\frac{1}{0.2}\right)^{\frac{1}{4}} = 5^{\frac{1}{4}} = \sqrt[4]{5}$ ۱۰