۱۸,۷۵

نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۴ ..... کلاس ۲۱۸.....

۱

$y=3^{Ax+B}$ ، $y=x^2 \to x=1,3 \Rightarrow y=1,9$

$x=1, y=1 \longrightarrow 1=3^{A+B} \Rightarrow A+B=0 \Rightarrow A=-B$ / $x=3, y=9 \longrightarrow 9=3^{3A+B} \Rightarrow 3A+B=2, -3B+B=2, -2B=2$

$B=-1 \Rightarrow A=1 \to y=3^{x-1} \Rightarrow$ if $x=0 \to y=3^{-1}=\frac{1}{3}$ → برخورد با محور y

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$2^{x+3}=4^x+15 \to 2^x \times 2^3-(2^x)^2-15=0$ $\quad 2^x=t$

$t^2-8t+15=0 \Rightarrow \Delta=4, x=3,5 \Rightarrow$ حاصل $=3+5=8$

$t=3 \to x=\log_2^3$

$t=5 \to x=\log_2^5$

$x_1+x_2=\log_2^{15}$

۳

$(\log_{21}^3)^2+\log_{21}^{(147)} \times \log_{21}^{(1323)}$ / $147=3\times7^2$, $1323=3^3\times7^2$ / $\log_{21}^3=t$

$\log_{21}^{147}=\log_{21}^3+2\log_{21}^7=2-a$ ← $\log_{21}^{21}=1 \Rightarrow \log_{21}^3+\log_{21}^7=1$, $\log_{21}^7=1-a$ / $\log_{21}^{1323}=2\log_{21}^3+2\log_{21}^7=3a+2(1-a)=a+2$

$\Rightarrow a^2+(a+2)(2-a)=a^2+4-a^2=4$

۴

$\log(1-x)^2+3\log(1-x)=5 \to 5\log(1-x)=5 \Rightarrow \log 1-x=1$

$10=1-x \Rightarrow x=-9 \Rightarrow \log_3^9=2$

۵

$\log_2^{x^2+2x+4}+\log_2^{x-2}=\log_2^8 \to (x^2+2x+4)\times(x-2)=8 \Rightarrow x^3+2x^2+4x-2x^2-4x-8$

$\Rightarrow x^3-8=8 \Rightarrow x^3=16, x=\sqrt[3]{16}$

$\log_{\sqrt{2}}^{\sqrt[3]{16}}=4$

۶

$\log(2-x) - \log\left(\frac{1}{(x-2)^2}\right) = 3 \rightarrow \log(2-x)(x-2)^2 = 3$ $D_f: 2-x>0 \Rightarrow x<2$

if $x<2 \Rightarrow (x-2)^2 = (2-x)^2 \Rightarrow \log(2-x)^3 = 3 \rightarrow (2-x)^3 = 10^3$

$\Rightarrow 2-x=10$, $x=-8$, $\log_{\sqrt{2}}^{(-x)} = ? \rightarrow \log_{\sqrt{2}}^{8} = 6$

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$3^{x^2-2} \leq 81^x \rightarrow 3^{x^2-2} \leq 3^{4x} \Rightarrow x^2-2 \leq 4x$, $x^2-4x-2 \leq 0 \Rightarrow \Delta = 24$

$x = \frac{4 \pm \sqrt{24}}{2} \Rightarrow x = 2+\sqrt{6}$, $\underbrace{2-\sqrt{6}}_{<0}$ $\log_4^{x-2} = \log_4^{2+\sqrt{6}-2} \Rightarrow \log_4^{\sqrt{6}} = \frac{1}{2}$

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$\log_3^2 = \frac{a}{8}$

$\log_{18}^{8} = \frac{\log_3^8}{\log_3^{18}} = \frac{3\log_3^2}{\log_3^2 + \log_3^9} = \frac{3 \times \frac{a}{8}}{2+\frac{a}{8}} = \frac{\frac{3a}{8}}{\frac{16+a}{8}} = \frac{3a}{16+a}$

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$\log_4^3 = 0.8$

$\log_{12}^{4} = \frac{\log_4^4}{\log_4^{12}} = \frac{\log_4^4}{\log_4^3 + \log_4^4} = \frac{0.8+0.5}{1+0.8} = \frac{1.3}{1.8} = \frac{13}{18}$

۱۰

$(a\log 2)(-1)^2 - a + b\log 2 = 0 \rightarrow a\log 2 + b\log 2 = a \rightarrow (a+b)\log 2 = a$

$\left(\frac{a+b}{a}\right)\log 2 = \frac{a}{a} \rightarrow \left(1+\frac{b}{a}\right)\log 2 = 1$, $1+\frac{b}{a} = \frac{1}{\log 2} \Rightarrow \frac{b}{a} = \underbrace{\frac{1}{\log 2}}_{\log_2^{10}} - 1 = \log_2^{10} - 1$

$(\sqrt{2})^{\frac{b}{a}} = 2^{\frac{b}{2a}} \rightarrow 2^{\frac{1}{2}(\log_2^{10}-1)} = 2^{\log_4^{10}} \times 2^{-\frac{1}{2}} = 10^{-\frac{1}{2}} \times \frac{1}{\sqrt{2}} = \sqrt{10} \times \frac{1}{\sqrt{2}} = \sqrt{5}$