۲۰

سارا ساسانی - یازدهم دختر A - تکلیف شماره ۲۴

-۱

$$\left.\begin{array}{l} y = x^2 \\ f(x) = 3^{Ax+B} \end{array}\right\} \xrightarrow{1,3} \begin{array}{l} 1 = 3^{A+B} \Rightarrow A+B=0 \\ 9 = 3^{3A+B} \Rightarrow 3A+B=2 \end{array}$$

$$\begin{array}{r} 3A+B=2 \\ -A-B=0 \\ \hline 2A=2 \Rightarrow A=1,\ B=-1 \end{array}$$

$$f(x) = 3^{x-1} \Rightarrow 3^{0-1} = 3^{-1} = \boxed{\frac{1}{3}}$$

-۲

$$\log_2(4^x+15) = x+3 \Rightarrow 2^{x+3} = 4^x+15 \Rightarrow 2^x \times 2^3 = 4^x + 15 \xrightarrow{\div 2^x}$$

$$2^3 = \underbrace{\frac{4^x}{2^x}}_{(\frac{4}{2})^x} + \frac{15}{2^x} \Rightarrow 8 = 2^x + \frac{15}{2^x} \xrightarrow{2^x = A} 8 = A + \frac{15}{A} \Rightarrow A^2 - 8A + 15 = 0$$

$$\left.\begin{array}{l} A=3 \to 2^x = 3 \to x = \log_2 3 \\ A=5 \to 2^x = 5 \to x = \log_2 5 \end{array}\right\} \text{جمع} = \log_2 3 + \log_2 5 = \boxed{\log_2 15}$$

-۳

$$(\log_{21} 3)^2 + \log_{21} 147 \cdot \log_{21} 1323 \longrightarrow \log_{21} 147 = 2\log_{21} 21 - \log_{21} 3$$

$$\log_{21} 1323 = 2\log_{21} 21 + \log_{21} 3 \Rightarrow (\cancel{2\log_{21} 21}^{\,2} - \log_{21} 3)(\cancel{2\log_{21} 21}^{\,2} + \log_{21} 3) = 4 - (\log_{21} 3)^2$$

$$(\log_{21} 3)^2 + 4 - (\log_{21} 3)^2 = \boxed{4}$$

-۴

$$x^2 - 2x + 1 = (x-1)^2 \Rightarrow (x-1)^2 \underbrace{(1-x)^3}_{-(x-1)} = 5 \Rightarrow -(x-1)^5 = 10^5 \Rightarrow$$

$$5 = \log_{10} 10^5$$

$$-x+1 = 10 \Rightarrow x = -9$$

$$\log_3 9 = \boxed{2}$$

-۵

$$3 = \log_2 8 \Rightarrow \underbrace{(x^2+2x+4)(x-2)}_{x^3 \cancel{-2x^2} \cancel{+2x^2} \cancel{-4x} \cancel{+4x} - 8} = 8 \Rightarrow x^3 = 16 \Rightarrow x = \sqrt[3]{16}$$

$$\log_{\sqrt[3]{2}} \sqrt[3]{16} = \boxed{4}$$

-۶

$-(x-۲)$

$۳=\log_{۱۰}^{۱۰^۳} \Rightarrow \dfrac{(۲-x)}{\frac{۱}{(x-۲)^۲}} = ۱۰^۳ \Rightarrow -(x-۲)^۳ = ۱۰^۳ \Rightarrow -x+۲=۱۰$

$\boxed{x=-۸}$

$\log_{\sqrt{۲}}^{۸} = \log_{۲^{\frac{۱}{۲}}}^{۲^۳} = ۶\log_۲^۲ = \boxed{۶}$

د

-۷

$۳^{x^۲-۲} = ۸۱^x \Rightarrow ۳^{x^۲-۲} = ۳^{۴x} \Rightarrow x^۲-۲=۴x \Rightarrow x^۲-۴x-۲=۰$

$x=\dfrac{۴\pm ۲\sqrt{۶}}{۲}$ → $۲-\sqrt{۶}$ غ‌ق‌ق → $۲+\sqrt{۶}$

$\log_۶^{(۲+\sqrt{۶}-۲)} = \log_۶^{\sqrt{۶}} = \boxed{\dfrac{۱}{۲}}$

د

-۸

$\log_{۱۸}^{۸} = \dfrac{\log_۳^۸}{\log_۳^{۱۸}} = \dfrac{۳\cancel{\log_۳^۲}^{\frac{۵}{۸}}}{\underbrace{\cancel{\log_۳^۹}}_{۲}+\underbrace{\cancel{\log_۳^۲}}_{\frac{۵}{۸}}} = \dfrac{\frac{۱۵}{۸}}{\underbrace{\cancel{۲+\frac{۵}{۸}}}_{\frac{۲۱}{۸}}} = \boxed{\dfrac{۵}{۷}}$

د

-۹

$\log_{۱۲}^{۶} = \dfrac{\log_۴^۶}{\log_۴^{۱۲}} = \dfrac{\log_۴^۲ + \cancel{\log_۴^۳}^{۰/۸}}{\underbrace{\cancel{\log_۴^۳}}_{۰/۸} + \underbrace{\cancel{\log_۴^۴}}_{۱}} = \dfrac{\frac{۱}{۲}\log_۲^۲ + ۰/۸}{۰/۸+۱} = \dfrac{۰/۵+۰/۸}{۱/۸} = \boxed{\dfrac{۱۳}{۱۸}}$

د

-۱۰

$(a\log ۲)x^۲ + ax + b\log ۲ = ۰ \xrightarrow{x=-۱} a\log ۲ - a + b\log ۲ = ۰ \Rightarrow$

$b\log ۲ = a - a\log ۲ \Rightarrow b\log ۲ = a(۱-\log ۲) \Rightarrow \dfrac{b}{a} = \dfrac{۱-\log ۲}{\log ۲} = \log_۲^۵$

$(\sqrt{۲})^{\log_۲^۵} = ۲^{\frac{۱}{۲}\log_۲^۵} = ۲^{\log_۲^{\sqrt{۵}}} = \sqrt{۵}^{\log_۲^۲} = \boxed{\sqrt{۵}}$

د