یکتا بخاری
مقطع : یازدهم دختر A

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(۱)

$y = x^2 \xrightarrow{x=1} y=1$, $\searrow x=3$, $y=9$

$f(x) = 3^{Ax+B} \quad (1,1) \rightarrow 1 = 3^{A+B} \rightarrow A+B=0$

$\searrow (3,9) = 9 = 3^{3A+B} \rightarrow 3A+B=2$

$\boxed{A=1}$ $\boxed{B=-1}$

$f^{-1}(x) = 3^{x-1} \xrightarrow{x=0} \boxed{\frac{1}{3}}$

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(۲)

$\log_2(4^x+15) = x+3 \rightarrow 2^{x+3} = 4^x+15 \rightarrow 2^x \times 8 = (2^x)^2+15$ ($2^x = t$)

$t^2 - 8t + 15 \rightarrow (t-3)(t-5)$

$t=5 \quad t=3$

$2^x = 5 \quad 2^x = 3$

$x_1 = \log_2 5 \quad x_2 = \log_2 3$

$x_1 + x_2 = \log_2 5 + \log_2 3 \Rightarrow \boxed{\log_2 15}$

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(۳)

$(\log_{21} 3)^2 + \log_{21}(147)\log_{21}(1323) \qquad \log_{21} 7 = 1 - \log_{21} 3$

$A = (\log_{21} 3)^2 + (\log_{21} 21 + \log_{21} 7)(\log_{21} 21^2 + \log_{21} 3) \rightarrow A = (\log_{21} 3)^2 + (1+\log_{21} 7)(2\log_{21} 21 + \log_{21} 3)$

$(\log_{21} 3)^2 + (1+1-\log_{21} 3)(2+\log_{21} 3) = (4-(\log_{21} 3)^2) + (\log_{21} 3)^2 = \underline{4}$

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(۴)

$\log(x^2-2x+1) + 3\log(1-x) = 5 \rightarrow \log(1-x)^2 + 3\log(1-x) - 5 = 0 \rightarrow 5\log_{10}(1-x) = -5$

$0/1 = 1-x \rightarrow \boxed{x = 0/9}$

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(۵)

$\log_2(x^2+2x+4) + \log_2(x-2) = 3 \qquad (x^2+2x+4)(x-2) = 8$

$x^3 - 8 = 8 \rightarrow x^3 = 16 \qquad \log_{2^{\frac{1}{3}}} x = 3\log_2 x =$

$\log_2 x^3 = \log_2 16 = \underline{4}$

(6)

$\log^{(2-x)} - \log_{(x-2)^2}^{1} = 3 \to \log^{(2-x)} - \log^{(2-x)^{-2}} = 3 \to \log^{2-x} + 2\log^{2-x} = 3$

$3\log^{2-x} = 3 \to x = -8$

$\log_{\sqrt2}^{8} = 6$

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(7)

if: $3^{x^2-2} = 81^x \to 3^{x^2-2} = 3^{4x} \to x^2 - 4x = 2$

$(x-2)^2 - 4 = 2 \to (x-2)^2 = 6 \to x-2 = \pm\sqrt6$ $\begin{cases} x = 2-\sqrt6 < 2 \ \times \\ x = 2+\sqrt6 \end{cases}$

$\log_6^{x-2} \xrightarrow{x=\sqrt6+2} \log_6^{2+\sqrt6-2} = \frac{1}{2}$

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(8)

$\log_3^2 = \frac{5}{8}$ $\quad \log_{18}^{8} = ?$ $\to \frac{\log_3^8}{\log_3^{18}} \to \frac{3\log_3^2}{2\log_3^3 + \log_3^2} \to \frac{\frac{15}{8}}{\frac{16+5}{8}} \to \frac{15}{21} = \frac{5}{7}$

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(9)

$\log_4^3 = 0.8 \to \frac{1}{2}\log_2^3 = 0.8 \to \log_2^3 = 1.6$

$\log_{12}^{6} \to \frac{\log_2^6}{\log_2^{12}} = \frac{\log_2^2 + \log_2^3}{2\log_2^2 + \log_2^3} \to \frac{1+1.6}{2+1.6} \to \frac{2.6}{3.6} \to \frac{1.3}{1.8}$

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(10)

$(a\log 2)x^2 + ax + b\log 2 = 0$

$\log 2\,(a+b) = a$ $\quad \frac{a+b}{a} \times \log 2 = 1 \to (1+\frac{b}{a})\log 2 = 1$

$\log 2^{(1+\frac{b}{a})} = 1 \to 10 = 2 \times 2^{\frac{b}{a}} \to 2^{\frac{b}{a}} = 5 \to (\sqrt2)^{\frac{b}{a}} =$

$2^{(\frac12)(\frac{b}{a})} = (2^{\frac{b}{a}})^{\frac12} \to 5^{\frac12} = \sqrt5$