نام و نام خانوادگی ...حسین یلدرمی‌خواه... پاسخنامه تشریحی تکلیف شماره ..۲۴.. کلاس ...یازدهم (A)...

۱

$y = x^2 \begin{cases} x=1 \to y=1 \\ x=3 \to y=9 \end{cases}$ $\qquad f(x) = 3^{Ax+B} \to f(0) = 3^{B} = 3^{-1} = \frac{1}{3}$

$f(x) = 3^{Ax+B} \begin{cases} x=1 \to f(x)=1 \to 3^{A+B}=1 \to A+B=0 \\ x=3 \to f(x)=3 \to 3^{3A+B}=9 \to 3A+B=2 \end{cases} \to A=1,\ B=-1$

۲

$\log_2 (4^x+15) = x+3 \to 2^{x+3} = 4^x+15 \Rightarrow 8(2^x) = (2^x)^2+15$

$2^x = t \to t^2-8t+15=0 \begin{cases} t=3 \\ t=5 \end{cases}$

$2^x=3 \to x=\log_2 3$ $\qquad 4^x+15>0$ همواره برقرار

$2^x=5 \to x=\log_2 5$ $\qquad 2^{(x+3)} > 2^{2x} \to x<3$

→ هر دو جواب قابل قبول

۳

$(\log_{21} 3)^2 + \log_{21} 147 \times \log_{21} 1323 = ?$

$(\log_{21}3 \times \log_{21}3) + (\log_{21}3 + \log_{21}7 + \log_{21}7)(\log_{21}3 + \log_{21}3 + \log_{21}3 + \log_{21}7 + \log_{21}7)$

$(\log_{21}3 \times \log_{21}3) + (1+\log_{21}7)(2+\log_{21}3) = (\log_{21}3 \times \log_{21}3) + 2 + 2\log_{21}3 + 2\log_{21}7 + (\log_{21}7 \times \log_{21}3)$

$= (\log_{21}3 \times \log_{21}3) + (\log_{21}7 \times \log_{21}3) + 4 = \log_{21}3(\log_{21}3 + \log_{21}7) + 4 = \log_{21}3 + 4$

۴

$\log(x^2-2x+1) + 3\log(1-x) = 5$ $\qquad \log_3 -x = \log_3 -(-9) = \log_3 9 = 2$

$2\log(x-1) + 3\log(-1(x-1)) = 5$

$\log((x-1)^5(-1)) = 5 \to (x-1)^5(-1) = 10^5$

توان فرد → $(x-1)^5 = (-10)^5 \Rightarrow x-1 = -10 \Rightarrow x = -9$ → به ازای -9 $\begin{cases} x^2-2x+1>0 \\ 1-x>0 \end{cases}$

۵

$\log_2 (x^2+2x+4) + \log_2 (x-2) = 3 \to (x-2)(x^2+2x+4) = 8$

$x^3-(2^3) = 8 \to x^3-8=8 \Rightarrow x^3=16 \Rightarrow x=\sqrt[3]{16}$

$\log_{\sqrt[3]{2}} x = \log_{\sqrt[3]{2}} \sqrt[3]{16} = \frac{3}{3}\log_2 16 = 1\times 4 = 4$

6

$\log^{(2-x)} - \log^{\frac{1}{(x-2)^2}} = 3 \longrightarrow \log^{(2-x)(x-2)^2} = 3 \longrightarrow (2-x)(x-2)^2 = 1000$

$(x-2)^3 = -1000$

$\sqrt[3]{(x-2)^3} = \sqrt[3]{-1000}$

$x-2 = -10$

$x = -8$

$0<2-x \xleftarrow[-8]{\text{برقرار}}$

$\log_{\sqrt{2}}^{-(x)} = \log_{\sqrt{2}}^{-(-8)} = 2\log_2^8 = 2\times 3 = 6$

7

$3^{x^2-2} = 81^x \longrightarrow 3^{x^2-2} = 3^{4x} \longrightarrow x^2-2 = 4x \quad x^2-4x-2=0 \begin{cases} x = 2+\sqrt{2} \\ x = 2-\sqrt{2} \end{cases}$

$\log_6^{(x-2)} = ? \xrightarrow{x=2+\sqrt{2}} x-2 = \sqrt{2}$ قابل قبول

$\xrightarrow{x=2-\sqrt{2}} x-2 = -\sqrt{2}$ غیرقابل قبول $\longrightarrow \log_6^{(x-2)} = \log_6^{\sqrt{2}}$

$\log_6^{\sqrt{2}} = \frac{1}{2}\log_6^2 = \frac{1}{2}\times\frac{1}{\log_2^6} = \frac{1}{2}\times\frac{1}{\log_2^2+\log_2^3} = \frac{1}{2}\times\frac{1}{1+\log_2^3} = \frac{1}{2+2\log_2^3}$

8

$\log_3^2 = \frac{5}{8}$

$\log_{18}^8 = 3\log_{18}^2 = \frac{3}{\log_2^{18}} = \frac{3}{\log_2^9+\log_2^2} = \frac{3}{2\log_2^3+1} =$

$\hookrightarrow \log_2^3 = \frac{8}{5} \longrightarrow \frac{3}{2(\frac{8}{5})+1} = \frac{3}{\frac{16}{5}+1} = \frac{3}{\frac{21}{5}} = 3\times\frac{5}{21} = \frac{5}{7}$

9

$\log_4^3 = \frac{8}{10}$

$\log_{12}^6 = \log_{12}^2 + \log_{12}^3 = \frac{1}{\log_2^{12}} + \frac{1}{\log_3^{12}} =$

$= \frac{1}{\log_2^4+\log_2^3} + \frac{1}{\log_3^4+\log_3^3} = \frac{1}{2+\log_2^3} + \frac{1}{1+\log_3^4} = \frac{1}{2+2\log_4^3} + \frac{1}{1+\log_3^4}$

$\hookrightarrow \log_3^4 = \frac{10}{8} \longrightarrow \frac{1}{2+2(\frac{8}{10})} + \frac{1}{1+\frac{10}{8}} = \frac{1}{2+\frac{8}{5}} + \frac{1}{1+\frac{10}{8}} = \frac{1}{\frac{18}{5}} + \frac{1}{\frac{18}{8}} = \frac{5+8}{18} = \frac{13}{18}$

10

$(a\log^2)x^2 + ax + b\log^2 = 0 \qquad x_1 = -1 \longrightarrow x_1\times x_2 = \frac{-b\log^2}{a\log^2} \rightarrow -x_2 = \frac{-b\log^2}{a\log^2}$

ریشه دوم معادله $\boxed{x_2 = \frac{b}{a}}$

$x_1 = -1 \longrightarrow x_1 + x_2 = \frac{-a}{a\log^2} \rightarrow -1+x_2 = \frac{-a}{a\log^2} \hookrightarrow x_2 = 1-\frac{a}{a\log^2} = 1-\log_2^{10}$

$(\sqrt{2})^{\frac{b}{a}} = (\sqrt{2})^{1-\log_2^{10}} = (\sqrt{2})^{\log_2^{\frac{2}{10}}} = (\frac{2}{10})^{\log_2^{\sqrt{2}}} = (\frac{2}{10})^{\frac{1}{2}} = \sqrt{\frac{2}{10}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}} = \boxed{\frac{\sqrt{5}}{5}}$