محمدمهدی میرمقتدایی ج

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$$x^2 = 3^{Ax+B} \Rightarrow x = 1, 3$$

$$\Rightarrow 3^2 = 3^{3A+B} \Rightarrow 2 = 3A+B$$

$$\Rightarrow 1 = 3^{A+B} \Rightarrow (0 = A+B) \times -1$$

$$\Bigg\} \Rightarrow 2A = 2 \Rightarrow A = 1, \quad B = -1$$

$$f(x) = 3^{x-1} \Rightarrow \boxed{f(0) = \frac{1}{3} \Rightarrow \left(0, \frac{1}{3}\right)}$$

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$$\log_2 (4^x + 15) = x+3 \rightarrow 4^x + 15 = 2^{x+3} \Rightarrow$$

$$2^{2x} + 15 = 8 \times 2^x \xrightarrow{2^x = t} t^2 - 8t + 15 = 0$$

$$(t-3)(t-5) = 0 \Rightarrow t = 3, 5 \Rightarrow 3 = 2^x \Rightarrow \log_2 3 = x$$

$$5 = 2^x \Rightarrow \log_2 5 = x$$

$$\log_2 3 + \log_2 5 = \boxed{\log_2 15}$$

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۳-

$$(\log_{21} 3)^2 + (2\underbrace{\log_{21} 7}_{1-m} + \underbrace{\log_{21} 3}_{m}) \times (2\log_{21} 7 + 3\log_{21} 3)$$

$$\log_{21} 7 = \log_{21} 21 - \log_{21} 3 \Rightarrow \log_{21} 7 = 1-m$$

$$\Rightarrow x^{۲}+\underbrace{\underbrace{(۲-۲x+x)}_{۲-x}\times\underbrace{(۲-۲x+۳x)}_{x+۲}}_{۴-x^{۲}}=x^{۲}-x^{۲}+۴=\boxed{۴}$$

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$(۱-x)^{۲}$

$$\log(x^{۲}-۲x+۱)+۳\log(۱-x)=۵$$ ۴←

$$۲\log(۱-x)+۳\log(۱-x)=۵\Rightarrow \log(۱-x)=۱\Rightarrow ۱-x=۱۰\Rightarrow$$

$$x=-۹\Rightarrow \log_{۳}(-(-۹))=\boxed{۲}$$

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$$\log_{۲}(x^{۲}+۲x+۴)+\log_{۲}(x-۲)=۳$$ ۵←

$$(x^{۲}+۲x+۴)(x-۲)=x^{۳}+\cancel{۲x^{۲}}+\cancel{۴x}-\cancel{۲x^{۲}}-\cancel{۴x}-۸=x^{۳}-۸ \Rightarrow$$

$$\log_{۲}(x^{۳}-۸)=۳\Rightarrow x^{۳}-۸=۸\Rightarrow x^{۳}=۱۶\Rightarrow x=۲^{\frac{۴}{۳}}$$

$$\log_{۲^{\frac{۱}{۳}}}۲^{\frac{۴}{۳}}=\frac{۴}{۳}\times۳\log_{۲}۲=\boxed{۴}$$

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$$\log(۲-x)-\log\frac{۱}{(x-۲)^{۲}}=۳\Rightarrow \log\left(-(x-۲)\div\frac{۱}{(x-۲)^{۲}}\right)=۳$$ ۶←

$$\log(-(x-۲)^{۳})=۳\Rightarrow -(x-۲)^{۳}=۱۰^{۳}\Rightarrow x-۲=-۱۰$$

$$x=-۸=-۲^{۳}\Rightarrow \log_{۲^{\frac{۱}{۲}}}۲^{۳}=۳\times۲\log_{۲}۲=\boxed{۶}$$

$$3^{x^2-2} = 3^{4x} \Rightarrow x^2-2 = 4x \Rightarrow x^2-4x-2=0$$ ←۷

$$\Delta = 16+8 = 24 \Rightarrow \frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6} \rightarrow 2+\sqrt{6}$$

غ ق ق ← چون منفی است

$$\log_6^{2+\sqrt{6}-2} = \log_6^{6^{\frac{1}{2}}} = \frac{1}{2}\log_6^{6} = \boxed{1/2}$$

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$$\log_3^{2} = 5/8$$ ←۸

$$\log_{18}^{8} = \frac{\log_3^{8}}{\log_3^{18}} = \frac{\frac{15}{8}}{\frac{21}{8}} = \frac{15}{21} = \boxed{\frac{5}{7}}$$

$$\log_3^{8} \rightarrow 3\log_3^{2} \rightarrow 3\times\frac{5}{8}$$

$$\log_3^{18} \rightarrow \log_3^{9} + \log_3^{2} = \frac{16}{8} + \frac{5}{8}$$

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$$\log_4^{3} = 0/8$$ ←۹

$$\log_{12}^{6} \Rightarrow \frac{\log_4^{6}}{\log_4^{12}} = \frac{0/8+0/5}{1/8} = \boxed{\frac{13}{18}}$$

$$\log_4^{6} \rightarrow \log_4^{3} + \log_4^{2}, \quad \log_4^{3} = 0/8, \quad \log_4^{2} \rightarrow \frac{1}{2}\log_2^{2} = 0/5$$

$$\log_4^{12} \rightarrow \log_4^{4} + \log_4^{3} = 1 + 0/8$$

$\xrightarrow{-1} a\log 2 - a + b\log 3 = 0$ ۱۵ $\leftarrow$

$b\log 3 = a - a\log 2 \rightarrow b\log 3 = a(1-\log 2)$

$b/a = \dfrac{1-\log 2}{\log 3} = \dfrac{\log 10 - \log 2}{\log 3} = \dfrac{\log 5}{\log 3} = \log_3 5$

$b/a = \log_3 5 \rightarrow (\sqrt{3})^{\log_3 5} = 5^{\log_3 \sqrt{3}} = 5^{\frac{1}{2}\log_3 3}$

$= 5^{1/2} = \boxed{\sqrt{5}}$