۶

$\log^{(x-2)} - \log_{(x-2)^2}^{\frac{1}{10}} = 3 \rightarrow \frac{2x}{\frac{1}{(x-2)^2}} = 10^3 \rightarrow (x-2)^2 \times (x-2)^{x-1} = 10^3 \rightarrow (x-2)^{x+1} = 10^3$

$x-2 = -10 \rightsquigarrow x = -1$

$\log_{\sqrt{2}}^{-x} \frac{1}{2} = \log_{\sqrt{2}}^{1} = 2 \log_2^8 = 2 \times 3 = 6$

۷

$3^{x^2-2} = 81^x \rightarrow 3^{x^2-2} = 3^{4x} \rightarrow x^2-2 = 4x \rightarrow x^2-4x-2 = 0$

$x = 2 \pm \sqrt{6}$

$\sqrt{2+\sqrt{6}} \quad x > 2$

$\times \; 2-\sqrt{6}$

$\log_6^{(x+2)} = \log_6^{(4+\sqrt{6}-2)} = \log_6^{\sqrt{6}} = \frac{1}{2}$

۸

$\log_2^2 = \frac{5}{8} \rightsquigarrow \frac{1}{\log_2^2} = \frac{5}{8} \rightarrow \log_{10}^{100} = \frac{8}{5}$

$\log_{10}^{8} = \frac{\log_2^8}{\log_2^{10}} = \frac{3}{\log_2^1 + \log_2^5} = \frac{3}{1+2\log_2^5} = \frac{3}{1+\frac{8}{5}\times 2} = \frac{3}{1+\frac{16}{5}} = \frac{\frac{15}{5}}{\frac{21}{5}} = \frac{5}{7}$

۹

$\log_4^2 = \frac{8}{10} \rightarrow \log_2^2 = \frac{10}{8} \rightarrow 2\log_2^2 = \frac{10}{8} \rightarrow \log_2^2 = \frac{10}{16} = \frac{5}{8}$

$\log_{12}^{6} = \frac{\log_2^6}{\log_2^{12}} = \frac{\log_2^{2\times 3}}{\log_2^{2^2\times 3}} = \frac{\log_2^2 + \log_2^3}{\log_2^{2^2} + \log_2^3} = \frac{1+\frac{5}{8}}{1+\frac{10}{8}} = \frac{\frac{13}{8}}{\frac{18}{8}} = \frac{13}{18}$

۱۰

$(a\log 2)x^2 + ax + b\log 2 = 0 \xrightarrow{x=-1} a\log 2 - a + b\log 2 = 0 \rightarrow (a+b)\log 2 = a$

$\rightarrow (1+\frac{b}{a})\log 2 = 1 \rightarrow 1+\frac{b}{a} = \frac{1}{\log 2} = \log_2^{10} \rightarrow \frac{b}{a} = \log_2^{10} - 1$

$\frac{b}{a} = \log_2^{10} - \log_2^2 + 1 = \log_2^5 - 1 + 1 \rightarrow \frac{b}{a} = \log 2^5 \Rightarrow \sqrt{2}^{\frac{b}{a}} = \sqrt{2}^{\log 2^5}$

$\rightarrow 5\log_2^{\sqrt{2}} = 5^{\frac{1}{2}} = \sqrt{5}$

نام و نام خانوادگی: بهار شهدادی — تکلیف ۲۴ — گروه یازدهم دختر A

۱)
$f(x)=3^{Ax+B}$ ، $y=x^2$ ، $x_1=1 , x_2=3$

$x=1 \to y=1 \Rightarrow x=1 \to f(x)=1=3^{A+B} \to A+B=0$
$x=3 \to y=9 \Rightarrow x=3 \to f(x)=9=3^{3A+B} \to 3A+B=2$ } $2A=2 \to A=1$ ، $B=-1$

$f(x)=3^{x-1} \to x=0 \Rightarrow 3^{-1}=\boxed{\frac{1}{3}}$ ← نقطه‌ی تلاقی تابع f با محور yها

۲)
$\log_2(4^x+15)=x+3 \to \log_2(4^x+15)=\log_2 2^{(x+3)}$ } $\to 2^x\times 2^3$

$2^x=t \to t^2+15=t\times 8 \to t^2-8t+15=0 \to (t-3)(t-5)=0$ ، $t=5$ ، $t=3$

$\to 2^x=5 \to x_1=\log_2 5$
$\to 2^x=3 \to x_2=\log_2 3$ ، جمع جواب‌ها: $\log_2 3+\log_2 5=\boxed{\log_2 15}$

۳)
$(\log_{21} 3)^2+\log_{21}(147)\,\log_{21} 1323$ ($147=7^2\times 3$ ، $1323=3^3\times 7^2$)

① $\log_{21}147=\log_{21}21+\log_{21}7=1+(\log_{21}21-\log_{21}3)=2-\log_{21}3$
② $\log_{21}1323=2\log_{21}21+\log_{21}3=2+\log_{21}3$

$\to (\log_{21}3)^2+(2-\log_{21}3)\times(2+\log_{21}3)=(\log_{21}3)^2+4-(\log_{21}3)^2=\boxed{4}$

۴)
$\log(x^2-2x+1)+3\log(1-x)=\log 10^5$

$(x^2-2x+1)\times(1-x)^3=10^5 \to (x-1)^2\times(x-1)^3=10^5 \to -(x-1)=10 \to x-1=-10$

$\boxed{x=-9}$ $\log_3 9=\boxed{2}$

۵)
$\log_2(x^2+2x+4)+\log_2(x-2)=3$ ($3=\log_2 8$)

$\to (x^2+2x+4)(x-2)=8 \to (x+2)^2(x-2)=8 \to x^3-8=8 \to x^3=16 \to x=\sqrt[3]{16}$

$\log_{\sqrt[3]{2}} x=\frac{\frac{1}{3}}{\frac{1}{3}}\log_2 16=\boxed{4}$