(۱)

$f(x) = 3^{Ax+B}$, $y = x^2$ $\longrightarrow$ $3^{A+B} = 1$, $3^A \times 3^B = 1$

$3^{3A+B} = 9$, $3^{3A} \times 3^B = 9$

$\frac{3^{3A} \times 3^B}{3^A \times 3^B} = 9 \Rightarrow \frac{(3^A)^3}{3^A} = 9 \overset{3^A = t}{\Rightarrow} \frac{t^3}{t} = t^2 = 9 \rightarrow t = \pm 3 \rightarrow 3^A = 3$

$\rightarrow A = -1$

$f(x) = 3^{x-1} \overset{x=0}{\Rightarrow}$ $\boxed{f(x) = \frac{1}{3}}$

(۲)

$\log_2 (4^x + 15) = x + 3 \Rightarrow 4^x + 15 = 2^{x+3} \Rightarrow 2^{2x} + 15 = 2^x \times 8$

$2^x = t \longrightarrow t^2 - 8t + 15 = 0 \Rightarrow (t-3)(t-5) = 0$

$2^x = 3 \Rightarrow \log_2 3 = t_1$

$2^x = 5 \Rightarrow \log_2 5 = t_2$

$t_1 + t_2 = \log_2 15$

(۳)

$(\log_{21} 3)^2 + \log_{21} 147 \cdot \log_{21} 1323$

$147 \rightarrow 7^2 \times 3$, $1323 \rightarrow 3 \times 7 \times 3 \times 7 \times 3$

$\log_{21} 3 = z$

$2y + z = 147 \longrightarrow 2(1-z) + z = 2 - 2z + z = 2 - z$

$3z + 2y = 1323 \longrightarrow 3z + 2(1-z) = 3z + 2 - 2z = z + 2$

$\log_{21} 7 = y$

$y + z = 1 \rightarrow y = 1 - z$

$z^2 + (2+z)(2-z) = z^2 + 4 - z^2 = \boxed{4}$

(۴)

$\log (x^2 - 2x + 1) + 3 \log (1-x) = 5 \Rightarrow \log (x-1)^2 + \log -(x-1)^3 = 5$

$\log -(x-1)^5 = 5 \Rightarrow 5 \log -(x-1) = 5 \Rightarrow -x + 1 = 10 \Rightarrow x = -9$

$\boxed{\log_3 9 = 2}$

(۵)

$\log_2 (x^2 + 2x + 4) + \log_2 (x - 2) = 3 \Rightarrow \log_2 (x-2)(x^2 + 2x + 4) = 3$

$x^3 + 2x^2 + 4x - 2x^2 - 4x - 8 = 8 \Rightarrow x^3 = 16 \Rightarrow x = \sqrt[3]{16}$

$\boxed{\log_{\sqrt[3]{4}} \sqrt[3]{16} = 2}$

$$\log^{-(x-1)} - \log^{\frac{1}{(x-1)^2}} = 1$$

$$\log^{-(x-1)(x-1)^2} = 1 \Rightarrow 2\log^{2-x} = 1 \Rightarrow 2-x = 1 \Rightarrow x = -1$$

$$\log^{\wedge}_{\sqrt{2}} 4 = 4$$

$$2^{x^2-2x} = 2^{3x} \Rightarrow x^2 - 2 = 3x \Rightarrow x^2 - 3x - 2 = 0 \rightarrow \frac{3+\sqrt{9}}{2-\sqrt{9}}$$

$$\log^{(2+\sqrt{9}-2)}_{9} = \log^{\sqrt{9}}_{9} = \boxed{\frac{1}{2}}$$

$$\log^{2}_{2} = \frac{3}{8} \Rightarrow \log^{2}_{2} = \frac{1}{3}$$

$$\log^{12}_{18} = \log^{4}_{18} + \log^{3}_{18} = \log^{4}_{18} + \frac{1}{3} = \frac{3}{3}\log^{\frac{8}{3}}_{18} + \frac{1}{3} = \frac{1}{12} = \frac{1}{6}$$

$$\log^{18}_{15} = \frac{5}{6}$$

$$\log^{2}_{4} = \frac{1}{2}\log^{2}_{2} = 0.15 \Rightarrow \log^{2}_{2} = \frac{3}{5}$$

$$\log^{12}_{4} = \log^{4}_{4} + \log^{3}_{4} = 1 + \frac{1}{\log^{4}_{3}} = 1 + \frac{1}{\log^{2}_{3} + \log^{2}_{3}} = 1 + \frac{1}{1+\frac{2}{5}}$$

$$1 + \frac{5}{12} = \frac{17}{12} \Rightarrow \log^{4}_{12} = \frac{12}{17}$$

$$(a\log 2)x^2 + ax + b\log 2 = -1 \Rightarrow x = -1 \quad a\log 2 - a + b\log 2 = 0$$

$$b\log^{2} + a\log^{2} = a \Rightarrow \log^{2}(b+a) = a \Rightarrow \log 2 = \frac{a}{b+a}$$

$$\frac{b+a}{a} \cdot \log 2 = 1, \quad 1 \Rightarrow \log^{2}_{\frac{b}{a}+1} = 1 \Rightarrow 10 = 2^{\frac{b}{a}} \times 2 \Rightarrow 2^{\frac{b}{a}} = 5$$

$$\boxed{(\sqrt{2})^{\frac{b}{a}} = \sqrt{5}}$$