ریاضیات (یازدهم تجربی تک سؤالی A)

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1)

$f(1)=g(1) \Rightarrow 3^{A+B} = 1 = 3^0 \Rightarrow A+B=0$

$f(3)=g(3) \Rightarrow 3^{3A+B} = 9 = 3^2 \Rightarrow 3A+B=2$

$\Rightarrow A=1 , B=1$

2

پس: $f(x)=3^{x-1} \Rightarrow f(0)=3^{-1} = \boxed{\frac{1}{3}}$

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2)

$\log_2(4^x+15) = x+3 \Rightarrow 4^x+15=2^{x+3} \Rightarrow (2^2)^x+15 = 2^x \times 2^3$

$\Rightarrow (2^x)^2+15 = 8\times 2^x \xrightarrow{2^x=t} t^2+15=8t \Rightarrow t^2-8t+15=0$

2

$\Rightarrow (t-3)(t-5)=0 \Rightarrow \begin{cases} t=3 \Rightarrow 2^x=3 \Rightarrow \log_2^{2^x}=\log_2^3 \Rightarrow x=\log_2^3 \\ t=5 \Rightarrow 2^x=5 \Rightarrow \log_2^{2^x} = \log_2^5 \Rightarrow x=\log_2^5 \end{cases}$

$\Rightarrow$ مجموع جواب‌ها $= \log_2^3+\log_2^5 = \boxed{\log_2^{15}}$

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3)

$1323 = 21^2\times 3 \quad 147=21\times 7 \Rightarrow A=(\log_{21}^3)^2 + \log_{21}^{(21\times 7)} \times \log_{21}^{(21^2\times 3)}$

$\Rightarrow A=(\log_{21}^3)^2 + (\log_{21}^{21}+\log_{21}^7)(\log_{21}^{21^2}+\log_{21}^3)$

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$\Rightarrow A=(\log_{21}^3)^2+(1+\log_{21}^7)\times(2\log_{21}^{21}+\log_{21}^3) \Rightarrow \log_{21}^3+\log_{21}^7=\log_{21}^{21} \Rightarrow \log_7=1-\log^3$

$A=(\log_{21}^3)^2+(1+1-\log_{21}^3)\times(2+\log_{21}^3) = (\log_{21}^3)^2+(4-(\log_{21}^3)^2) \Rightarrow A=(\log_{21}^3)^2+4-(\log_{21}^3)^2 = \boxed{4}$

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4)

$\log(x^2-2x+1)+3\log(1-x)=5 \Rightarrow \log(x-1)^2 + \log(1-x)^3 = 5$

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$\log(1-x)^2 + \log(1-x)^3 = 5 \Rightarrow \log (1-x)^2\times(1-x)^3 = 5 \Rightarrow \log (1-x)^5 = 5 \Rightarrow (1-x)^5 = 10^5$

$\Rightarrow 1-x=10 \Rightarrow x=-9 \qquad \log_3^{(-x)} \Rightarrow \log_3^{(-(-9))} = \log_3^9 = \boxed{2}$

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5)

$\log_2(x^2+2x+4)+\log_2(x-2)=3 \qquad \log_{\sqrt[3]{2}}^x=?$

$\Rightarrow \log_2 (x^2+2x+4)(x-2) = 3 \Rightarrow \log_2 x^3-8 = 3 \Rightarrow x^3-8=2^3 \Rightarrow x^3=16 \Rightarrow x=\sqrt[3]{16}$

2

$\Rightarrow \operatorname{Log}_{\sqrt[3]{2}}^{\sqrt[3]{16}} = \operatorname{Log}_2^{16} = \boxed{4}$

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6)

$\log(2-x) - \log \frac{1}{(x-2)^2} = 3 \Rightarrow \log(2-x) - \log \frac{1}{(2-x)^2} = 3 \Rightarrow \log \frac{(2-x)}{\frac{1}{(2-x)^2}} = 3$

2

$\Rightarrow \log (2-x)^3 = 3 \Rightarrow 10^3 = (2-x)^3 \xrightarrow{\sqrt[3]{}} 10 = 2-x \Rightarrow x=-8$

$\log_{\sqrt{2}}^{(-x)} \Rightarrow \log_{\sqrt{2}}^8 = \log_{2^{\frac{1}{2}}}^{2^3} = 6 \log_2^2 = \boxed{6}$

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7)

$\log_6^{(x-2)}=? \qquad 3^{x^2-2} = 81^x \Rightarrow 3^{x^2-2}=3^{4x} \Rightarrow x^2-2=4x \Rightarrow x^2-4x-2=0$

$\Rightarrow \sqrt{\Delta} = \sqrt{b^2-4ac} = \sqrt{16+8} = \sqrt{24} = 2\sqrt{6} \qquad x = \frac{4\pm 2\sqrt{6}}{2}$ غ ق ق

$\Rightarrow x=2+\sqrt{6}$

2

$\Rightarrow \log_6(2+\sqrt{6}-2) = \log_6^{\sqrt{6}} = \log_6^{6^{\frac{1}{2}}} = \frac{1}{2}\log_6^6 = \boxed{\frac{1}{2}}$

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$\log_{۳}^{۲} = \frac{۵}{۸} \qquad \log_{۱۸}^{۸} = ?$

$$\Rightarrow \frac{\log_{۳}^{۸}}{\log_{۳}^{۱۸}} = \frac{\log_{۳}^{۲^{۳}}}{\log_{۳}^{۲} + \log_{۳}^{۹}} = \frac{۳\log_{۳}^{۲}}{\log_{۳}^{۲} + ۲} = \frac{۳ \times \frac{۵}{۸}}{\frac{۵}{۸} + ۲} = \boxed{\frac{۵}{۷}}$$

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$\log_{۴}^{۳} = ۰/۸$

$$\log_{۱۲}^{۶} = ? \Rightarrow \frac{\log_{۴}^{۶}}{\log_{۴}^{۱۲}} = \frac{\log_{۴}^{۳} + \log_{۴}^{۲}}{\log_{۴}^{۳} + \log_{۴}^{۴}} = \frac{۰/۸ + \frac{۱}{۲}}{۰/۸ + ۱} = \frac{\frac{۱۳}{\cancel{۱۰}}}{\frac{۱۸}{\cancel{۱۰}}} = \boxed{\frac{۱۳}{۱۸}}$$

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۱۰

$(a\log ۲)x^{۲} + ax + b\log ۲ = ۰$

$$x = -۱ \xRightarrow{ax^{۲}+bx+c=۰} a + c = b \Rightarrow a\log ۲ + b\log ۲ = a \Rightarrow b\log ۲ = a - a\log ۲$$

$$(\sqrt{۲})^{\frac{b}{a}} = ? \qquad \Rightarrow b\log ۲ = a(۱ - \log ۲) \Rightarrow \frac{b}{a} = \frac{۱ - \log ۲}{\log ۲}$$

$$\Rightarrow \frac{\log ۱۰ - \log ۲}{\log ۲} = \frac{\log \frac{۱۰}{۲}}{\log ۲} = \frac{\log ۵}{\log ۲} = \log_{۲}^{۵}$$

$$\Rightarrow (\sqrt{۲})^{\frac{b}{a}} = (\sqrt{۲})^{\log_{۲}^{۵}} = (۵)^{\log_{۲}^{\sqrt{۲}}} = ۵^{\frac{۱}{۲}\log_{۲}^{۲}} = ۵^{\frac{۱}{۲}} = \boxed{\sqrt{۵}}$$

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