پرنیا جانگیری - یازدهم دفتر A - تکلیف ۲۴

(۱)

$3^{Ax+B} = x^2$

$x=1:\ 3^{A+B} = 1 \Rightarrow A+B=0 \Rightarrow B=-A=-1$

$x=3:\ 3^{3A+B} = 3^2 \Rightarrow 3A+B=2 \Rightarrow 2A=2 \Rightarrow A=1$ (با $B=-A$)

$f(x) = 3^{x-1} \xRightarrow{x=0} 3^{-1} = \frac{1}{3}$

x=0 قرار دادیم تا نقاط برخورد با محور yها را بدست آوریم

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(۲)

$\log_2(4^x+15) = x+3 \Rightarrow 4^x + 15 = 2^{x+3} = 2^x \times 8 \Rightarrow (2^x)^2 - 8(2^x) + 15 = 0$

($4^x = (2^x)^2$)

$t^2 - 8t + 15 = 0$, $(t-5)(t-3)$

$t=3 \Rightarrow 2^x = 3 \Rightarrow \log_2 3 = x$

$t=5 \Rightarrow 2^x = 5 \Rightarrow \log_2 5 = x$

$\log_2 3 + \log_2 5 = \log_2 15$

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(۳)

$(\log_{21} 3)^2 + \log_{21} 147 \cdot \log_{21} 1323 \Rightarrow (\log_{21} 3)^2 - (\log_{21} 3)^2 + 4 = 4$

$147 = 7 \times 21$

$1323 = 3 \times 7 \times 63 = 21^2 \times 3$

$\log_{21} 147 = \log_{21} 21 + \log_{21} 7 = 1 + \log_{21} 7 = 1 + \log_{21} 21 - \log_{21} 3 = 2 - \log_{21} 3$ ($7 = \frac{21}{3}$)

$\log_{21} 1323 = 2\log_{21} 21 + \log_{21} 3 = 2 + \log_{21} 3$

$\Rightarrow (2+\log_{21} 3)(2-\log_{21} 3) = 4 - (\log_{21} 3)^2$

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(۴)

$\log(x^2-2x+1) + 3\log(1-x) = 5 \Rightarrow 5\log(1-x) = 5 \Rightarrow \log(1-x) = 1 \Rightarrow 1-x = 10 \Rightarrow x = -9$

$\log(1-x)^2 = 2\log(1-x)$

جواب نهایی: $\log_3(-x) = \log_3 9 = 2$

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(۵)

$\log_2(x^2+2x+4) + \log_2(x-2) = 3$

$\log_2 (x-2)(x^2+2x+4) = 3 \Rightarrow \log_2(x^3-8) = 3 \Rightarrow x^3 - 8 = 8 \Rightarrow x^3 = 16 \Rightarrow \sqrt[3]{16} = \sqrt[3]{2^4}$

$\log_{\sqrt[3]{2}} x = \log_{2^{\frac{1}{3}}} 2^{\frac{4}{3}} = \frac{\frac{4}{3}}{\frac{1}{3}} \log_2 2 = 4$

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(۶)

$\log(2-x) - \log\frac{1}{(x-2)^2} = 3 \Rightarrow \log(2-x) + 2\log(2-x) = 3 \Rightarrow \log(2-x) = 1 \Rightarrow 2-x = 10 \Rightarrow x = -8$

$\log\frac{1}{(x-2)^2} = \log\frac{1}{(2-x)^2} = \log 1 - 2\log(2-x) = -2\log(2-x)$

جواب نهایی: $\log_{2^{\frac{1}{2}}} (-x) = \log_{2^{\frac{1}{2}}} 2^3 = \frac{3}{\frac{1}{2}} \log_2 2 = 6$

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(۷)

$3^{4x} = 3^{x^2-2} \Rightarrow 4x = x^2 - 2 \Rightarrow x^2 - 4x - 2 = 0 \Rightarrow x^2 - 4x + 4 - 4 - 2 = 0 \Rightarrow (x-2)^2 - 6 = 0 \Rightarrow (x-2)^2 = 6 \Rightarrow x - 2 = \sqrt{6}$

$\log_6(x-2) = \log_6 \sqrt{6} = \frac{1}{2}$

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(۸)

$\log_{18} 8 = 3\log_{3^2 \times 2} 2 = 3 \times \frac{\log_3 2}{\log_3 (3^2 \times 2)} = 3 \times \frac{\frac{5}{8}}{2\log_3 3 + \log_3 2} = 3 \times \frac{\frac{5}{8}}{\frac{21}{8}} = 3 \times \frac{5}{21} = \frac{5}{7}$

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(۹)

$\log_{12} 6 = \frac{\log_4 6}{\log_4 12} = \frac{\log_4 2 + \log_4 3}{\log_4 4 + \log_4 3} = \frac{\frac{1}{2} + \frac{8}{10}}{1 + \frac{8}{10}} = \frac{\frac{13}{10}}{\frac{18}{10}} = \frac{13}{18}$

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(۱۰)

$(a\log 2)x^2 + ax + b\log 2 = 0 \xRightarrow{x=-1} a\log 2 + b\log 2 = a$

$(a+b)\log 2 = a \Rightarrow \log 2 = \frac{a}{a+b} \Rightarrow \frac{1}{\log 2} = \frac{a+b}{a} = 1 + \frac{b}{a} \Rightarrow \frac{1}{\log 2} - 1 = \frac{b}{a}$

$\frac{\log 10 - \log 2}{\log 2} = \frac{b}{a} \Rightarrow \frac{\log 5}{\log 2} = \frac{b}{a} \Rightarrow \log_2 5 = \frac{b}{a}$

$\log_2 5 = \frac{b}{a} \Rightarrow (2^{\frac{1}{2}})^{\log_2 5} = 5^{\frac{1}{2}} = \sqrt{5}$