سوال ۱)

$f(x)=3^{Ax+B}$, $y=x^2$ — محل تلاقی ۲ نمودار به ازای $x=1$ و $x=3$

$3^{Ax+B}=x^2 \longrightarrow \log_3^{x^2}=Ax+B \rightarrow 2\log_3^x=Ax+B$

$x=1 \longrightarrow 2\log_3^1=A+B \rightarrow A+B=0$

$x=3 \longrightarrow 2\log_3^3=3A+B \rightarrow 3A+B=2$

$-A-B=0$

$2A=2 \rightarrow A=1 \longrightarrow B=-1$

$f(x)=3^{x-1}$

عرض تقاطع تلاقی با محور y ها $\Rightarrow x=0 \longrightarrow f(0)=3^{-1}=\frac{1}{3}$

سوال ۲)

$\log_2^{4^x+15}=x+3 \longrightarrow 4^x+15=2^{x+3} \longrightarrow 2^{2x}-2^{x+3}+15=0$

$(2^x)^2-(2^x\times 2^3)+15=0 \longrightarrow t^2-8t+15=0$

$t=2^x=3 \longrightarrow \log_2^3=x_1$

$t=2^x=5 \longrightarrow \log_2^5=x_2$

$x_1+x_2=\log_2^3+\log_2^5=\log_2^{15}$

سوال ۳)

$(\log_{21}^3)^2+\log_{21}^{147}\times\log_{21}^{1323}=(\log_{21}^3)^2+\log_{21}^{21\times7}\times\log_{21}^{21^2\times3}$

$(\log_{21}^3)^2+(\log_{21}^{21}+\log_{21}^7)(\log_{21}^{21^2}+\log_{21}^3)$

$\log_{21}^7=\log_{21}^{\frac{21}{3}}=\log_{21}^{21}-\log_{21}^3=1-\log_{21}^3$

$\longrightarrow (\log_{21}^3)^2+(2-\log_{21}^3)(2+\log_{21}^3)$

$\Rightarrow (\log_{21}^3)^2+4-(\log_{21}^3)^2=4$

سوال ۴)

$\log\underbrace{(x^2-2x+1)}_{(1-x)^2}+3\log(1-x)=5 \longrightarrow \log(1-x)^2+3\log(1-x)=5$

$\rightarrow 2\log(1-x)+3\log(1-x)=5 \rightarrow 5\log(1-x)=5 \rightarrow \log(1-x)=1 \rightarrow 1-x=10 \rightarrow x=-9$

$\log_3^{-(-9)}=\log_3^9=2$

سوال ۵)

$\log_2^{(x^2+2x+4)}+\log_2^{(x-2)}=3 \longrightarrow \log_2^{(x-2)(x^2+2x+4)}=3 \longrightarrow \log_2^{x^3-8}=3 \rightarrow x^3-8=8$

$x^3=16 \rightarrow x=2\sqrt[3]{2}$

$\log_{\sqrt[3]{2}}^{x} \Rightarrow \log_{\sqrt[3]{2}}^{2\sqrt[3]{2}}=4$

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سوال ۶)

$$\log(2-x) - \log\frac{1}{\frac{(x-2)^2}{(2-x)^2}} = 3 \rightarrow \log(2-x) - \log\frac{1}{(2-x)^2} = 3 \longrightarrow \log\frac{2-x}{\frac{1}{(2-x)^2}} = 3 \rightarrow \log(2-x)^3 = 3$$

$$(2-x)^3 = 10^3 \longrightarrow 2-x = 10 \rightarrow \boxed{x = -8}$$

$$\log_{\sqrt{2}}(-x) \Rightarrow \log_{\sqrt{2}}^{-(-8)} = \log_{\sqrt{2}}^{8} = 6$$

سوال ۷)

$$3^{x^2-2} = \underbrace{81}_{3^4}^{x} \rightarrow 3^{x^2-2} = 3^{4x} \rightarrow x^2-2 = 4x \rightarrow x^2-4x-2=0 \rightarrow x^2-4x+4-6=0$$

$$\rightarrow (x-2)^2-6=0 \rightarrow (x-2)^2 = 6 \longrightarrow x-2 = \sqrt{6} \qquad \log_4^{x-2} \Rightarrow \log_4^{\sqrt{6}} = \frac{1}{2}$$

سوال ۸)

$$\log_{18}^{8} = \log_{18}^{2^3} = 3\log_{18}^{2} = \frac{3}{\log_2^{18}} = \frac{3}{\log_2^{3^2\times 2}} = \frac{3}{\log_2^{3^2}+\log_2^{2}} = \frac{3}{2\log_2^{3}+1}$$

$$\log_3^2 = \frac{5}{8} \longrightarrow \log_3^2 = \frac{1}{\log_2^3} = \frac{5}{8} \rightarrow \log_2^3 = \frac{8}{5} \rightarrow \frac{3}{\frac{16}{5}+1} = \frac{3}{\frac{21}{5}} = \frac{5}{7}$$

سوال ۹)

$$\log_4^3 = 0.1 \rightarrow \frac{1}{2}\log_2^3 = 0.1 \rightarrow \log_2^3 = 1.6$$

$$\log_{12}^{4} = \log_{3\times 2^2}^{3\times 2} = \frac{\log_2^{3\times 2}}{\log_2^{3\times 2^2}} = \frac{\log_2^3+\log_2^2}{\log_2^3+\log_2^{2^2}} = \frac{1.6+1}{1.6+2} = \frac{2.6}{3.6} = \frac{13}{18}$$

سوال ۱۰)

$$(a\log 2)x^2 + ax + b\log 2 = 0 \xrightarrow{x=-1} a\log 2 - a + b\log 2 = 0$$

$$(a+b)\log 2 = a \longrightarrow \log 2 = \frac{a}{a+b} \longrightarrow \frac{1}{\log 2} = \frac{a+b}{a} = 1+\frac{b}{a}$$

$$\rightarrow \frac{1}{\log 2} - 1 = \frac{b}{a} \qquad \frac{1-\log 2}{\log 2} = \frac{b}{a} \longrightarrow \frac{\log 10 - \log 2}{\log 2} = \frac{\log 5}{\log 2} = \log_2^5 = \frac{b}{a}$$

$$(\sqrt{2})^{\frac{b}{a}} \longrightarrow (\sqrt{2})^{\log_2^5} = 5^{\log_2^{\sqrt{2}}} = 5^{\frac{1}{2}} = \boxed{\sqrt{5}}$$