

$x=1 \Rightarrow f(1) = 3^{A(1)+B} = 3^{A+B}$   
 $3^{A+B} = 3 \Rightarrow A+B = 1$  ①  
 $x=3 \Rightarrow f(3) = 3^{3A+B} = 3$   
 $3^{3A+B} = 3 \Rightarrow 3A+B = 1$  ②

$① - ② = (3A+B) - (A+B) = 2 - 0$   
 $2A = 2 \Rightarrow A = 1, B = -1$   
 $f(0) = 3^{A(0)+B} = 3^{-1} = \frac{1}{3}$

$4^x + 15 = 2^{x+3}$   
 $2^{2x} + 15 = 2^{x+3} \Rightarrow 2^{2x} + 15 = 8 \times 2^x$   
 $t^2 + 15 = 8t \Rightarrow t^2 - 8t + 15 = 0$   
 $\Rightarrow t = 3, t = 5$   
 $2^x = 3 \Rightarrow \log_2 3$   
 $2^x = 5 \Rightarrow \log_2 5$   
 $\log_2 3 + \log_2 5 = \log_2 15$

$21 = 3 \times 7$      $147 = 3 \times 7^2$      $1323 = 3^3 \times 7^2$   
 $\log_{21} 147 = \log_{21} (3 \times 7^2) = \log_{21} 3 + 2 \log_{21} 7$   
 $\log_{21} 1323 = \log_{21} (3^3 \times 7^2) = 3 \log_{21} 3 + 2 \log_{21} 7$   
 $a = \log_{21} 3$      $b = \log_{21} 7$      $\log_{21} 21 = 1 \Rightarrow a + b = 1$   
 $a^2 + (a+2b)(3a+2b) \Rightarrow 3a+2-2a = a+2 \Rightarrow (2-a)(a+2) + a^2$   
 $2a(a+2) + (2-a)(a+2) = 4 - a^2 \Rightarrow a^2 + (4 - a^2) = 4$

$x^2 - 2x + 1 = (x-1)^2 \Rightarrow \log(x-1)^2 + 3 \log(1-x) = 5$   
 $2 \log(1-x) + 3 \log(1-x) = 5 \Rightarrow 5 \log(1-x) = 5$   
 $\log(1-x) = 1 \Rightarrow 1-x = 10 \Rightarrow x = -9$   
 $\log_3(-9) = \log_3 9 = 2$

$\log_2 [(x^2 + 2x + 4)(x-2)] = 3 \Rightarrow (x^2 + 2x + 4)(x-2) = 2^3 = 8$   
 $(x^2 + 2x + 4)(x-2) = 8 \Rightarrow x^3 - 8 = 8 \Rightarrow x^3 = 16 \Rightarrow x = \sqrt[3]{16}$   
 $\log_{\sqrt{2}} x \Rightarrow x = (2^4)^{1/3} = 2^{4/3}$   
 $\log_{\sqrt{2}} \frac{2^{4/3}}{2^{1/3}} = \frac{4/3}{1/3} = 4$

$$2 - a > 0 \rightarrow a < 2$$

.6

$$\log\left(\frac{2-a}{(a-2)^2}\right) = 3 \Rightarrow \log((2-a)(a-2)^2)$$

$$(a-2)^2 = (2-a)^2 \Rightarrow (2-a)(2-a)^2 = (2-a)^3$$

$$\log(2-a)^3 = 3 \Rightarrow 3 \log(2-a) = 3 \Rightarrow \log(2-a) = 1 \Rightarrow 2-a = 10 \Rightarrow a = -8$$

$$\log\frac{(-8)}{\sqrt{2}} \Rightarrow \log\frac{8}{\sqrt{2}} = \frac{3}{\frac{1}{2}} = 6$$

$$3^{a^2-2} = 3^{4a} \Rightarrow a^2-2 = 4a \Rightarrow a^2-4a-2=0 \Rightarrow a = 2 \pm \sqrt{6}$$

.7

~~Handwritten scribbles and crossed-out work.~~

$$a > 2 \Rightarrow a = 2 + \sqrt{6} > 2 \checkmark$$

$$\Rightarrow a - 2 = \sqrt{6} \quad 2 - \sqrt{6} < 2 \times$$

$$\log_6 6 = \frac{1}{2}$$

$$8 = 2^3 \quad 18 = 2 \times 3^2$$

.8

$$\log_{18} 8 = \frac{\log 8}{\log 18}$$

$$\log 8 = \log 2^3 = 3 \log 2$$

$$\log 18 = \log 2 + 2 \log 3$$

$$\frac{\log 2}{\log 3} = \frac{5}{8} \Rightarrow \log 2 = \frac{5}{8} \log 3 \Rightarrow \frac{5}{8} \log 3 + \frac{16}{3} \log 3 = \frac{21}{8} \log 3$$

$$3 \log 2 = 3 \times \frac{5}{8} \log 3 = \frac{15}{8} \log 3$$

$$\frac{\frac{15}{8} \log 3}{\frac{21}{8} \log 3} = \frac{5}{7}$$

$$\frac{\log 3}{\log 4} = \frac{4}{5} \Rightarrow \log 3 = \frac{4}{5} \log 4 \quad \log 3 = \frac{4}{5} \times 2 \log 2 = \frac{8}{5} \log 2$$

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$$\log 6 = \log 2 + \log 3 = \frac{13}{5} \log 2 \quad \log 12 = 2 \log 3 + \log 4 = \frac{18}{5} \log 2$$

$$\frac{\text{①}}{\text{②}} = \frac{13}{18}$$

$$(a \log 2)(-1)^2 + a(-1) + b \log 2 = 0 \Rightarrow a \log 2 - a + b \log 2 = 0$$

.10

$$(a+b) \log 2 - a = 0 \Rightarrow (a+b) \log 2 = a \quad a+b = \frac{a}{\log 2} \quad b = \frac{a}{\log 2} - a$$

$$b = a(\frac{1}{\log 2} - 1) \quad \frac{b}{a} = \frac{1}{\log 2} - 1 \quad (\sqrt{2})^{\frac{b}{a}} = 2^{\frac{1}{\log 2} - 1}$$

$$2^{\frac{1}{\log 2} - 1} = 10 \quad \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}$$