۱. $y = x^2 \xrightarrow{3,1} 9 = 3^2,\ 1 = 1 \Rightarrow 1 = 3^{A+B},\ 9 = 3^{3A+B} \Rightarrow 0 = A+B \Rightarrow B = -A$

$\Rightarrow 2 = 3A + B \Rightarrow 2 = 3A - A \Rightarrow 2 = 2A \Rightarrow A = 1$

$\Rightarrow F(x) = 3^{x-1} \Rightarrow 3^{0-1} = 3^{-1} \Rightarrow \boxed{\frac{1}{3}}$

(۲)

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۲. $2^{x+3} = 4^x + 15 \Rightarrow 8(2^x) = 4^x + 15 \Rightarrow 2^{2x} - 8(2^x) + 15 = 0$, $2^x = t$

$\Rightarrow t^2 - 8t + 15 = 0 \Rightarrow t = 3,\ t = 5$

(۲)

$\Rightarrow 2^x = 3 \Rightarrow \log_2 3$

$\Rightarrow 2^x = 5 \Rightarrow \log_2 5$

$\Rightarrow \log_2 5 + \log_2 3 = \boxed{\log_2 15}$

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۳. $\log_{21} 1323 = \log_{21} (21^2 \times 3) = \log_{21} 21^2 + \log_{21} 3 = 2 + \log_{21} 3$

$\log_{21} 147 = \log_{21} \frac{(21)^2}{3} = \log_{21} 21^2 - \log_{21} 3 = 2 - \log_{21} 3$

(۳)

$\Rightarrow (\log_{21} 3)^2 + (2 - \log_{21} 3)(2 + \log_{21} 3) = (\log_{21} 3)^2 + 4 - (\log_{21} 3)^2 = \boxed{4}$

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۴. $\log (1-x)^2 + 3\log (1-x) = 5 \Rightarrow 2\log(1-x) + 3\log(1-x) = 5$

$\Rightarrow 5\log(1-x) = 5 \Rightarrow \log(1-x) = 1 \Rightarrow 1 - x = 10 \Rightarrow x = -9$

(۲)

$\Rightarrow \log_3 (-x) = \log_3 9 = \boxed{2}$

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۵. $\log_2 (x^2+2x+4)(x-2) = 3 \Rightarrow \log_2 (x^3 - 8) = 3 \Rightarrow 8 = x^3 - 8 \Rightarrow x^3 = 16$, $x = \sqrt[3]{16}$

$\log_{\sqrt[3]{2}} \sqrt[3]{16} = \boxed{4}$

(۲)

-۶

$$\log_{\frac{1}{(2-x)^2}} \frac{2-x}{1} = 3 \Rightarrow \log (2-x)^3 = 3 \Rightarrow 10 = 2-x \Rightarrow x = -8$$

(۴)

$$\log_{\sqrt{2}} (-x) = \log_{\sqrt{2}} 8 = \frac{3}{\frac{1}{2}} \log_2 2 = \boxed{6}$$

-۷

$$3^{x^2-2} = 3^{4x} \Rightarrow x^2 - 4x - 2 = 0 \Rightarrow \begin{cases} x = \frac{4-2\sqrt{6}}{2} & \times \\ x = \frac{4+2\sqrt{6}}{2} & \checkmark \end{cases} \Rightarrow \log_6 (x-2)$$

(۴)

$$\log_6 (2+\sqrt{6}-2) = \log_6 \sqrt{6} = \boxed{\frac{1}{2}}$$

(۸

$$\frac{1}{\log_8 18} = \frac{1}{\log_8 2 + \log_8 9} = \frac{1}{\frac{1}{3} + \log_8 9} = \frac{1}{\frac{1}{3} + \frac{2}{3} \times \log_2 3} \Rightarrow \frac{1}{\frac{1}{3} + \frac{2}{3} \times \frac{8}{5}}$$

$$\frac{1}{\log_2 3} = \frac{5}{8} \Rightarrow \frac{8}{5} = \log_2 3$$

(۴)

$$\Rightarrow \frac{1}{\frac{21}{15}} = \frac{15}{21} = \boxed{\frac{5}{7}}$$

(۹

$$\log_{12} 3 + \log_{12} 2 = \frac{1}{\log_3 4 + \log_3 3} + \frac{1}{\log_2 4 + \log_2 3} = \frac{8}{18} + \frac{5}{18} = \boxed{\frac{13}{18}}$$

$$\left(\log_3 4 = \frac{10}{8},\ \log_3 3 = 1,\ \log_2 4 + \log_2 3 = 2 + 1.6\right)$$

(۴)

$$\log_4 3 = 0.8 \Rightarrow \frac{1}{\log_3 4} = 0.8 \Rightarrow \frac{1}{2} \log_2 3 = 0.8 \Rightarrow \log_2 3 = 1.6$$

$$\Rightarrow \log_3 4 = \frac{10}{8}$$

(۱۰

$$(a \log 2) - a + b \log 2 = 0 \Rightarrow b \log 2 = a - a \log 2$$

$$\Rightarrow b(\log 2) = a(1 - \log 2) \Rightarrow \frac{b}{a} = \frac{1 - \log 2}{\log 2}$$

(۴)

$$\Rightarrow (\sqrt{2})^{\frac{1}{\log 2} - 1} = \frac{1}{\sqrt{2}} \times (\sqrt{2})^{\log_2 10} = \frac{1}{\sqrt{2}} \times 10^{\log_2 \sqrt{2}} = \frac{1}{\sqrt{2}} \times \sqrt{10} = \boxed{\sqrt{5}}$$