۱.

$y = x^2 \xrightarrow{3,1} 9 = 3^2$, $1 = 1$ $\Rightarrow$ $1 = 3^{A+B}$, $9 = 3^{3A+B}$ $\Rightarrow$ $0 = A+B \Rightarrow B = -A$

$\Rightarrow 2 = 3A+B \Rightarrow 2 = 3A - A \Rightarrow 2 = 2A \Rightarrow A = 1$

$\Rightarrow F(x) = 3^{x-1} \Rightarrow 3^{0-1} = 3^{-1} \Rightarrow \boxed{\frac{1}{3}}$

۲.

$2^{x+3} = 4^x + 15 \Rightarrow 8(2^x) = 4^x + 15 \Rightarrow 2^{2x} - 8(2^x) + 15 = 0$, $2^x = t$

$\Rightarrow t^2 - 8t + 15 = 0 \Rightarrow t = 3$, $t = 5$

$\Rightarrow 2^x = 3 \Rightarrow \log_2 3$

$\Rightarrow 2^x = 5 \Rightarrow \log_2 5$

$\Rightarrow \log_2 3 + \log_2 5 = \boxed{\log_2 15}$

۳.

$\log_{21} 343 = \log_{21} 7^3 = 3 \times \log_{21} 7 = 3 \log_{21} 7 = 2 + \log_{21} 3$

$\log_{21} 147 = \log_{21} \frac{(21)^2}{3} = \log_{21} 21^2 - \log_{21} 3 = 2 - \log_{21} 3$

$\Rightarrow (\log_{21} 3)^2 + (2 - \log_{21} 3)(2 + \log_{21} 3) = \cancel{(\log_{21} 3)^2} + 4 - \cancel{(\log_{21} 3)^2} = \boxed{4}$

۴.

$\log (1-x)^2 + 3\log(1-x) = 5 \Rightarrow 2\log(1-x) + 3\log(1-x) = 5$

$\Rightarrow 5\log(1-x) = 5 \Rightarrow \log(1-x) = 1 \Rightarrow 1-x = 10$

$\Rightarrow x = -9$

$\Rightarrow \log_3(-x) = \log_3 9 = \boxed{2}$

۵.

$\log_2 (x^2+2x+4)(x-2) = 3 \Rightarrow \log_2 (x^3 - 8) = 3 \Rightarrow 8 = x^3 - 8 \Rightarrow x^3 = 16$

$x = \sqrt[3]{16}$

$\log_2 \sqrt[3]{16} = \boxed{4}$

۶-

$$\log \frac{\frac{2-x}{1}}{\frac{1}{(2-x)^2}} = 3 \Rightarrow \log (2-x)^3 = 3 \Rightarrow 10 = 2-x \Rightarrow x = -8$$

$$\log_{\sqrt{2}}^{(-x)} = \log_{\sqrt{2}}^{8} = \frac{3}{\frac{1}{2}} \log_2^2 = \boxed{6}$$

۷-

$$3^{x^2-2} = 3^{60} \Rightarrow x^2 - 4x - 2 = 0 \Rightarrow x = \frac{4-2\sqrt{6}}{2} \times$$

$$x = \frac{\cancel{4}^{2}+2\sqrt{6}}{\cancel{2}} \checkmark \Rightarrow \log_6^{(x-2)}$$

$$\log_6^{(2+\sqrt{6}-2)} = \log_6^{\sqrt{6}} = \boxed{\frac{1}{2}}$$

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۸)

$$\frac{1}{\log_8^{15}} = \frac{1}{\log_8^3 + \log_8^5} = \frac{1}{\frac{1}{3} + \log_8^5} = \frac{1}{\frac{1}{3} + \frac{2}{3} \times \log_2^5} = \frac{1}{\frac{1}{3} + \frac{2}{3} \times \frac{1}{\log_5^2}}$$

$$\frac{1}{\log_2^5} = \frac{5}{8} \Rightarrow \frac{8}{5} = \log_2^5 \qquad \Rightarrow \frac{1}{\frac{21}{15}} = \frac{15}{21} = \boxed{\frac{5}{7}}$$

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۹)

$$\log_{12}^3 + \log_{12}^2 = \frac{1}{\underbrace{\log_3^4 + \log_3^3}_{1}} + \frac{1}{\underbrace{\log_2^4 + \log_2^3}_{2}} = \frac{8}{18} + \frac{5}{18} = \boxed{\frac{13}{18}}$$

$$\frac{1}{\log_3^4} = 0.8 \Rightarrow \log_3^4 = \frac{10}{8}$$

$$\log_2^3 = 0.8 \Rightarrow \frac{1}{2}\log_2^3 = 0.8 \Rightarrow \log_2^3 = 1.6$$

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۱۰)

$$(a\log 2) - a + b\log 5 = 0 \Rightarrow b\log 5 = a - a\log 5$$

$$\Rightarrow b(\log 5) = a(1 - \log 5) = \frac{b}{a} = \frac{1-\log 5}{\log 5}$$

$$\Rightarrow (\sqrt{2})^{\frac{1}{\log 5} - 1} = \frac{1}{\sqrt{2}} \times (\sqrt{2})^{\log_2^{10}} = \frac{1}{\sqrt{2}} \times 10^{\log_2^{\sqrt{2}}} = \frac{1}{\sqrt{2}} \times \sqrt{10} = \boxed{\sqrt{5}}$$