سارا سامانی - یازدهم دختر A - تکلیف شماره ۲۵

۱-

$(1.5, 0) \to 1 - \log_c^{(-1.5a-b)} = 0 \Rightarrow \log_c^{(-1.5a-b)} = 1 \Rightarrow c^1 = -1.5a - b$

$\underbrace{b+c}_{-\frac{3}{2}} = -1.5a \Rightarrow -\frac{3}{2} = -\frac{3}{2}a \Rightarrow a = 1 \Rightarrow y = 1 - \log_c^{(x-b)}$

$(0,2) \to 1 - \log_c^{-b} = 2 \Rightarrow \log_c^{-b} = -1 \to \frac{1}{c} = -b \quad b + c = -\frac{3}{2} \Rightarrow$

$c + \frac{3}{2} = -b \Rightarrow c + \frac{3}{2} = \frac{1}{c} \Rightarrow c^2 + \frac{3}{2}c = 1 \to 2c^2 + 3c - 2 = 0$

$\Delta = 9 - 4(2)(-2) = 25 \quad \frac{-3 \pm 5}{4} \nearrow \frac{1}{2} \to b = -2$

$\searrow -2$ غ ق ق

$\boxed{(a+c)b = -3}$

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$(1,0) \to 1 + c \times 3^{a+b} = 0 \Rightarrow c \times 3^{a+b} = -1$

$(0, \frac{2}{3}) \to 1 + c \times 3^a = \frac{2}{3} \Rightarrow c \times 3^a = -\frac{1}{3}$ $\Big\}\ b = 1$

$f(-1) = 1 + c \times 3^{a-1} = 1 + \frac{c \times 3^a}{3} = 1 + \frac{-\frac{1}{3}}{3} = 1 - \frac{1}{9} = \boxed{\frac{8}{9}}$

۳-

$(0, 2.4) \to c + \log_5^{2.4a+b} = 0 \Rightarrow -c = \log_5^{2.4a+b} \Rightarrow 5^{-c} = 2.4a + b$

$(0,2) \to c + \log_5^b = 2 \to \log_5^b = 2 - c \Rightarrow 5^{2-c} = b \Rightarrow$

$5^{-c} - 5^{2-c} = 2.4a \Rightarrow 5^{-c}(\underbrace{\cancel{1-25}}_{\cancel{-24}\ -10}) = \cancel{2.4}a \Rightarrow a = -10 \times 5^{-c}$

$\frac{a}{b} = \frac{-10 \times \cancel{5^{-c}}}{25 \times \cancel{5^{-c}}} = \boxed{-0.4}$

۴-

$y = \log_4^{(|x^2-2|-x)} \Rightarrow |x^2-2| - x > 0 \Rightarrow |x^2-2| > x \Rightarrow x^2 - 2 > x$

$x^2 - 2 < -x \Rightarrow$

$x^2 - x - 2 > 0 \Rightarrow x > 2$ یا $x < -1 \quad \mathrm{I}$

$x^2 + x - 2 < 0 \Rightarrow x > -2$ یا $x < 1 \quad \mathrm{II}$ $\Big\}\ \mathrm{I} \cup \mathrm{II}\ \boxed{(-\infty, 1) \cup (2, +\infty)}$

5-

$$f(x)=2+2^{b-ax},\quad g(x)=-x^2-3x+8 \Big\} \; x=1 \to 2+2^{b-a}=4 \Rightarrow 2^{b-a}=2 \Rightarrow b-a=1$$

$$f^{-1}(10)=-1 \Rightarrow f(-1)=10 \Rightarrow 2+2^{b+a}=10 \Rightarrow 2^{b+a}=8 \Rightarrow b+\cancel{a}=3$$

$$b-\cancel{a}=1$$

$$b=2,\ a=1$$

$$2b-a=3$$

6-

$$f(x)=-2+\left(\frac{1}{2}\right)^{Ax+B},\quad y=x^2-x \Big\} \; x=1 \Rightarrow -2+\left(\frac{1}{2}\right)^{A+B}=0 \Rightarrow \left(\frac{1}{2}\right)^{A+B}=2 \Rightarrow A+B=-1$$

$$x=2 \Rightarrow -2+\left(\frac{1}{2}\right)^{2A+B}=2 \Rightarrow \left(\frac{1}{2}\right)^{2A+B}=4 \Rightarrow 2A+B=-2$$

$$2A+\cancel{B}=-2$$

$$-A-\cancel{B}=1$$

$$A=-1,\quad B=0$$

$$f(3)=-2+\left(\frac{1}{2}\right)^{-3}=6$$

7-

$$\underbrace{A_n}_{\text{مقدار باقی مانده}}=\left(\frac{8}{9}\right)^n \times \underbrace{A_0}_{\text{جرم اولیه}}$$

$$\frac{1}{6}\cancel{A_0}=\left(\frac{8}{9}\right)^n \times \cancel{A_0} \Rightarrow \frac{1}{6}=\left(\frac{8}{9}\right)^n \Rightarrow 6=\left(\frac{9}{8}\right)^n \xrightarrow{\log_2}$$

$$\log_2 5=2.4 \Rightarrow \log_5 2=\frac{1}{2.4}$$

$$\log_3 5=1.4 \Rightarrow \log_5 3=\frac{1}{1.4}$$

$$\Big\} \; \log_2 3=\frac{2.4}{1.4}=\frac{12}{7}$$

$$\log_2\left(\frac{9}{8}\right)^n=\log_2 6 \Rightarrow n\log_2\frac{9}{8}=\log_2 6 \Rightarrow n=\frac{\log_2 6}{\log_2\frac{9}{8}}=\frac{\underset{12/7}{\cancel{\log_2 3}}+\underset{1}{\cancel{\log_2 2}}}{\underset{\underset{24/7}{\cancel{2\log_2 3}}}{\cancel{\log_2 9}}-\underset{3}{\cancel{\log_2 8}}}=$$

$$\frac{1+\frac{12}{7}}{\frac{24}{7}-3}=\frac{19}{3}$$

8-

$$A_n=\left(\frac{7}{8}\right)^n \times A_0$$

$$\frac{1}{7}\times\cancel{A_0}=\left(\frac{7}{8}\right)^n \times \cancel{A_0} \Rightarrow \frac{1}{7}=\left(\frac{7}{8}\right)^n \Rightarrow 7=\underbrace{\left(\frac{8}{7}\right)^n}_{\log_3 \text{از طرفین}}$$

$$\log_7 3=0.6 \Rightarrow \log_3 7=\frac{10}{6}=\frac{5}{3}$$

$\log_2^3 = 1.6 \Rightarrow \log_3^2 = \frac{10}{16} = \frac{5}{8}$

$\cancel{\log_3^7} = \log_3\left(\frac{8}{7}\right)^n \Rightarrow \frac{10}{6} = n\log_3\frac{8}{7} \Rightarrow \frac{5}{3} = n(\log_3 8 - \log_3 7) \Rightarrow$

$\frac{10}{6}$

$\frac{5}{3} = n(3\underbrace{\log_3 2}_{\frac{5}{8}} - \underbrace{\log_3 7}_{\frac{5}{3}}) \Rightarrow \frac{5}{3} = n(\underbrace{\frac{15}{8} - \frac{5}{3}}) \Rightarrow \frac{5}{3} = n \times \cancel{\frac{5}{24}} \Rightarrow n = 8$

۹ـ

$(0.96)^n \times \cancel{A} = \frac{1}{3}\cancel{A} \Rightarrow (0.96)^n = \frac{1}{3} \Rightarrow \left(\frac{100}{96}\right)^n = 3 \xrightarrow{\log_{10}}$

$\log\left(\frac{100}{96}\right)^n = \underbrace{\log 3}_{0.48} \Rightarrow n\log\frac{96}{100} = 0.48 \Rightarrow n(\underbrace{\cancel{\log}100}_{2} - \log 96) = 0.48 \Rightarrow$

$n[2 - (\log 3 + \underbrace{\log 32}_{5\underbrace{\log 2}_{0.3}})] = 0.48 \Rightarrow n[2 - (1.5 + 0.48)] = 0.48 \Rightarrow$

$0.02n = 0.48 \Rightarrow n = 24$

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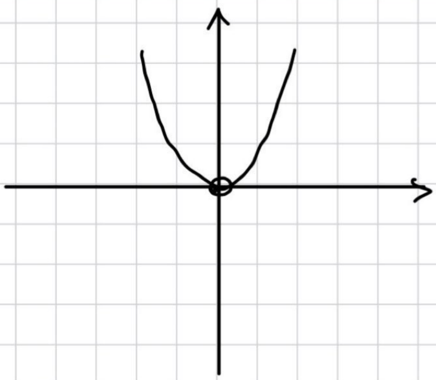
الف)

$y = 9^{\log_3 x} = x^{\log_3 9} = x^2$



ب)

$y = \log x^2$