

سوال (۱)

$$y = 1 - \log_c(ax-b) \rightarrow \log_c^c - \log_c^{ax-b} \rightarrow y = \log_c \frac{c}{ax-b}$$

$$(0, 2) \rightarrow \log_c \frac{-c}{b} = 2 \rightarrow \frac{-c}{b} = c^2 \rightarrow \frac{-1}{b} = c \rightarrow b = \frac{-1}{c}$$

$$\rightarrow b+c = \frac{-1}{c} \rightarrow \frac{-1}{c} + c = \frac{-1}{c} \rightarrow 2c^2 - 2 = -1 \rightarrow 2c^2 + 1c - 2 = 0 \rightarrow c^2 + 3c - 4 = 0$$

عقوت چون نمودار صعودی است $c > 1$

① $c = -1$
 $\rightarrow c = \frac{1}{4}$

$$b + \frac{-1}{c} = \frac{-1}{c} \rightarrow b = \frac{-1}{c}$$

$$(-1/a, 0) \rightarrow \log_{-1} \frac{-1}{-1/a + 1/c} = 0 \rightarrow \frac{-1}{-1/a + 1/c} = 1 \rightarrow -1/a + 1/c = -1 \rightarrow -1/a = -1 - 1/c \rightarrow a = 1$$

$$(a+c)b = (1-1) \frac{1}{c} = 0$$

سوال (۲)

$$(0, \frac{1}{9}) \rightarrow 1 + c \times r^a = \frac{1}{9} \rightarrow c \times r^a = -\frac{1}{9} \rightarrow \begin{cases} c = -1 \\ a = -1 \end{cases}$$

$$(1, 0) \rightarrow 1 + -1 \times r^{-1+b} = 0 \rightarrow b = 1$$

$$f(-1) = 1 + -1 \times r^{-1-1} = 1 - \frac{1}{9} = \frac{8}{9}$$

سوال (۳)

$$(0, 2) \rightarrow c + \log_a b = 2$$

$$(2, 0) \rightarrow c + \log_a \frac{1}{c} = 0$$

$$\log_a \frac{1}{c} - \log_a b = -2 \rightarrow \log_a \frac{1/c}{b} = -2 \rightarrow \frac{1/c}{b} = \frac{1}{c^2} \rightarrow \frac{1}{cb} = \frac{1}{c^2} \rightarrow c = b$$

$$\rightarrow 4 \cdot a + 2ab = b \rightarrow 4 \cdot a = -2ab \rightarrow \frac{a}{b} = \frac{-2c}{4} = -\frac{1}{2}$$

سوال (۴)

$$|x^2 - 2| - x > 0 \rightarrow |x^2 - 2| > x \rightarrow \begin{cases} x^2 - 2 > x \rightarrow x^2 - x - 2 > 0 \\ x^2 - 2 < -x \rightarrow x^2 + x - 2 < 0 \end{cases}$$

$\begin{matrix} \frac{-1}{2} & 2 \\ + & - & + \\ \hline \textcircled{1} \end{matrix}$
 $\begin{matrix} -2 & 1 \\ + & - & + \\ \hline \textcircled{2} \end{matrix}$

$$D_f = \textcircled{1} \cap \textcircled{2} = (-2, -1)$$

سوال (۵)

$$x=1 \rightarrow -(1)^2 - 2(1) + 1 = -2 \rightarrow$$

پس نقطه (۱، -۲) در هر دو تابع صدق میکند

$$(b, 4) \rightarrow 2 + 2^{b-a} = 4 \rightarrow 2^{b-a} = 2 \rightarrow b-a = 1$$

$$f^{-1}(1) = -1 \rightarrow f(1) = 1$$

$$(1, 1) \rightarrow 2 + 2^{b+a} = 1 \rightarrow 2^{b+a} = -1 \rightarrow 2^{b+a} = 2^{\log_2(-1)} \rightarrow b+a = \log_2(-1)$$

$$\begin{cases} b-a = 1 \\ b+a = 3 \\ \hline b=2 \quad a=1 \end{cases}$$

$$2b-a = 4-1 = 3$$

$y = x^2 - x \xrightarrow{x=1} 1-1=0 \rightarrow (1,0)$
 $\xrightarrow{x=2} 4-2=2 \rightarrow (2,2)$

این نقطه در تابع $f(x)$ صدق می‌کنند

$(1,0) \rightarrow -2 + \left(\frac{1}{2}\right)^{A+B} = 0 \rightarrow 2^{-A+B} = 2 \rightarrow -A-B = 1$
 $(2,2) \rightarrow -2 + \left(\frac{1}{2}\right)^{2A+B} = 2 \rightarrow 2^{-2A-B} = 4 \rightarrow 2^{-2A-B} = 2^2 \rightarrow -2A-B = 2$

$\begin{cases} -A-B=1 \\ -2A-B=2 \end{cases} \rightarrow \begin{matrix} -A-B=1 \\ -A=1 \rightarrow A=-1 \\ B=0 \end{matrix}$

$\rightarrow f(x) = -2 + \left(\frac{1}{2}\right)^{-x}$
 $f(3) = -2 + \left(\frac{1}{2}\right)^{-3} = -2 + 8 = 6$

$P \times \left(\frac{A}{9}\right)^t = \frac{1}{9} P \rightarrow \left(\frac{A}{9}\right)^t = \frac{1}{9} \rightarrow \log\left(\frac{A}{9}\right)^t = \log 9^{-1} \rightarrow t \log \frac{A}{9} = -\log 9$

$\log \frac{A}{9} = \log A^{\frac{1}{3}} - \log 9^{\frac{1}{3}} \rightarrow \frac{1}{3} \log A - \frac{1}{3} \log 9 = \frac{1}{3} \left(\frac{\log A}{3} - \frac{\log 9}{3}\right)$

$\log A = \frac{1}{\frac{\log 9}{3}} = \frac{1}{\frac{1}{3}} = 3$
 $\log 9 = \log 3^2 = 2 \log 3 = \frac{2}{3}$

$\frac{1}{3} \left(\frac{3}{3} - \frac{2}{3}\right) = \frac{1}{3} \left(\frac{1}{3}\right) = \frac{1}{9}$

$t \left(\frac{1}{9}\right) = -\log 9 = -\frac{2}{3} \rightarrow (t+1) \frac{1}{9} = (2t-1) \left(\frac{1}{3}\right)$

$\rightarrow 2t - 12 = 2t + 7 \rightarrow 2t = 19 \rightarrow t = \frac{19}{2} h = 9.5 \text{ min}$

$P \times \left(\frac{A \sqrt{A}}{1000}\right)^t = \frac{1}{10} P \rightarrow \log\left(\frac{A \sqrt{A}}{1000}\right)^t = \log 10^{-1} \rightarrow t \log \frac{A \sqrt{A}}{1000} = -\log 10$

$\log \frac{A \sqrt{A}}{1000} = \log A^{\frac{3}{2}} - \log 10^3 = \log A^{\frac{3}{2}} - \log 10^3 = \frac{3}{2} \log A - 3 \log 10$

$= \log A^{\frac{3}{2}} - 3 \log 10$

$t \left(\frac{3}{2} \log A - 3 \log 10\right) = -\log 10 \rightarrow t \left(\frac{3}{2}\right) - 3t \left(\frac{1}{10}\right) = -\frac{1}{10}$

$\log A = \frac{1}{\frac{\log 10}{3}} = \frac{1}{\frac{1}{3}} = 3$
 $\log 10 = \frac{1}{\frac{\log 10}{1}} = \frac{1}{1} = 1$

$(t+1) \frac{3}{2} = 3t \left(\frac{1}{10}\right) \rightarrow 9t = 18t + 18$

$\rightarrow t = 18 \text{ هفته} = 54 \text{ روز}$

اگر هر روز ۴ لیتر از محلول برداریم و جای آن آب بیزیم غلیظ آن $\frac{24}{25} = \frac{100-4}{100}$ برابر می‌شود

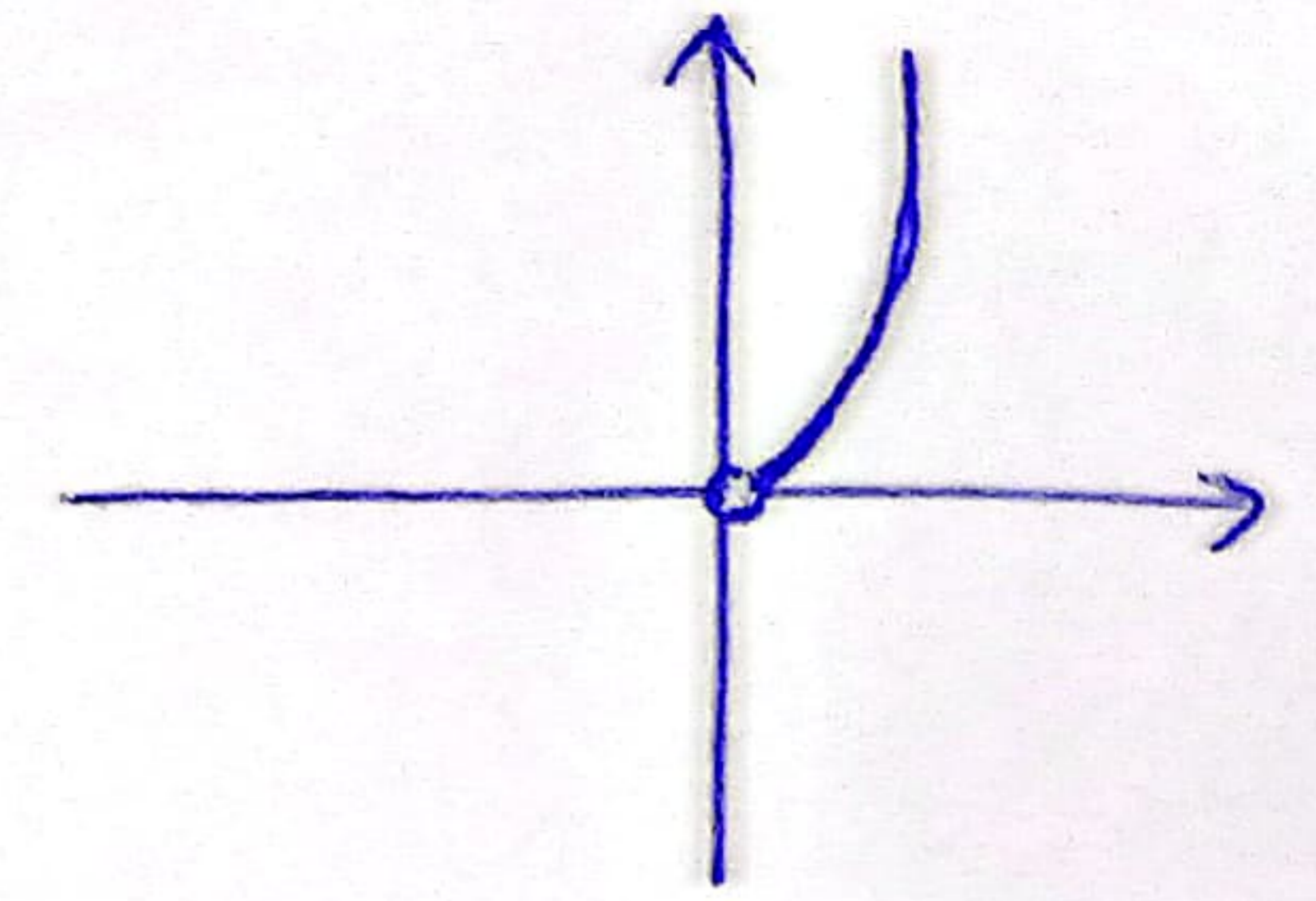
$100 \times \left(\frac{24}{25}\right)^t = \frac{1}{10} \times 100 \rightarrow \log\left(\frac{24}{25}\right)^t = \log \frac{1}{10} \rightarrow t \log \frac{24}{25} = -\log 10$

$\log \frac{24}{25} = \log 24^{\frac{1}{3}} - \log 25^{\frac{1}{3}} = \log 3 + \log 2 - \log 5 = \log 3 + \log 2 - \log 5 = \frac{1}{3} \log 3 + \frac{1}{3} \log 2 - \frac{1}{3} \log 5 = -0.104$

$\rightarrow t = \frac{-\log 10}{-0.104} = \frac{1}{0.104} = 9.615 \text{ روز}$

$D_f = (0, +\infty)$

الف) $y = 9^{\log_3 x} \rightarrow x^{\frac{\log_3 9}{3}} = y \rightarrow y = 2x^2$



ب) $y = \log_3 2x^2 = 2 \log_3 2x$

