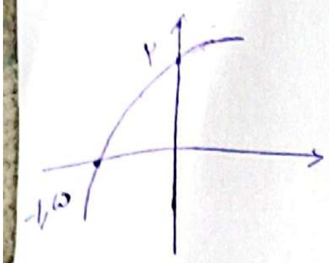
آزادکوهانی — پاسخنامه تشریحی شماره ۱۵ — یازدهم ریاضی A

۱)

$y = 1 - \log_c(ax-b)$



$b + c = -\frac{3}{2} \quad , \quad c - \frac{1}{c} = -\frac{3}{2} \Rightarrow c^2 - 1 + \frac{3}{2}c = 0 \Rightarrow 2c^2 - 2 + 3c = 0 \quad c = \frac{-3 \pm \sqrt{9+16}}{4}$

$\boxed{c = \frac{1}{2}} \quad c = -2$ غ ق ق

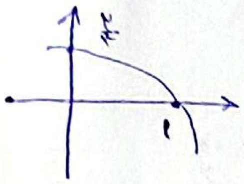
$x = 0 \Rightarrow 1 - \log_c^{-b} = 2 \Rightarrow \log_c^{-b} = -1 \Rightarrow -b = \frac{1}{c} \quad \boxed{b = -2}$

$y = 0 \Rightarrow 0 = 1 - \log_{\frac{1}{2}}(-1.5a+2) \Rightarrow 1 = \log_{\frac{1}{2}}^{-1.5a+2} \Rightarrow \frac{1}{2} = -1.5a + 2 \Rightarrow -1.5 = -1.5a \quad \boxed{a = 1}$

$(1 + \frac{1}{2})(-2) = \frac{-6}{2} = -3$

۲)

$f(x) = 1 + c \times 3^{a+bx}$



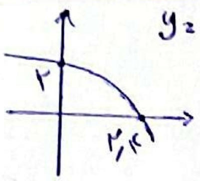
$x = 0 \Rightarrow \frac{2}{3} = 1 + c \times 3^a \Rightarrow -\frac{1}{3} = c \times 3^a$

$x = 1 \Rightarrow 0 = 1 + c \times 3^{a+b} \Rightarrow -1 = c \times 3^{a+b} \Rightarrow -1 = c \times 3^a \times 3^b \Rightarrow 3^b = 3 \Rightarrow b = 1$

$x = -1 \Rightarrow y = 1 + \underbrace{c \times 3^a}_{-\frac{1}{3}} \times \underbrace{3^{-b}}_{\frac{1}{3}} \Rightarrow y = 1 + (-\frac{1}{9}) \Rightarrow y = \frac{8}{9}$

۳)

$y = c + \log_5(ax+b)$



$x = 0 \Rightarrow 2 = c + \log_5 b$

$x = \frac{12}{5} \Rightarrow 0 = c + \log_5 \frac{12}{5}a + b$

$2 = \log_5 b - \log_5 \frac{12}{5}a + b \Rightarrow 2 = \log_5 \frac{b}{\frac{12}{5}a+b} \Rightarrow 25 = \frac{b}{\frac{12}{5}a+b} \Rightarrow b = 25b + 60a$

$-24b = 60a \Rightarrow \frac{a}{b} = \frac{-24}{60} = \frac{-4}{10} = \frac{-2}{5}$

۴)

$f(x) = \log_2(|x^2-2| - x)$

$|x^2-2| - x > 0 \Rightarrow |x^2-2| > x \begin{cases} x^2 - 2 > x \Rightarrow x^2 - x - 2 > 0 \\ -x^2 + 2 < x \Rightarrow 0 < x^2 - 2 + x \end{cases}$

$\begin{array}{c|c|c} & -1 & 2 & \\ + & - & + \end{array} \Rightarrow (-\infty, -1) \cup (2, +\infty)$

$\begin{array}{c|c|c} & -2 & 1 & \\ + & - & + \end{array} \Rightarrow (-\infty, -2) \cup (1, +\infty)$

$(-\infty, -1) \cup (2, +\infty) = D_f$

۵)

$f(x) = 2 + 2^{b-ax} \quad , \quad g(x) = -x^2 + 3x + 8$

$2 + 2^{b-a} = -1 + 3 + 8 \Rightarrow 8 = 2^{b-a} \Rightarrow b - a = 3$

$\underset{x=-1}{\Rightarrow} 2 + 2^{b+a} = 10 \Rightarrow 2^{b+a} = 8 \Rightarrow b + a = 3$

$b - a = 3$

$b + a = 3$

$2b = 6 \Rightarrow b = 3 \Rightarrow a = 0$

$2b - a = 6 - 0 = 6$

(9)

$x = 1 \Rightarrow \cancel{1-1}^{\,0} = -2 + \left(\frac{1}{2}\right)^{A+B} \Rightarrow \left(\frac{1}{2}\right)^{A+B} = 2 \Rightarrow A+B = -1$

$x = 2 \Rightarrow \cancel{4 - \frac{4}{2}} = -2 + \left(\frac{1}{2}\right)^{2A+B} \Rightarrow 4 = \left(\frac{1}{2}\right)^{2A+B} \Rightarrow 2A+B = -2$

$f(3) = -2 + \left(\frac{1}{2}\right)^{-3} = -2 + 8 = 6$

$A = -1 \Rightarrow B = 0$

(7)

$y = m\left(\frac{8}{9}\right)^t \Rightarrow \frac{1}{9}m = m\left(\frac{8}{9}\right)^t \Rightarrow \log_5^{\frac{1}{9}} = t\log_5^{\frac{8}{9}} \Rightarrow -\log_5^9 = t(\log_5^8 - \log_5^9)$

$-(\log_5^2 + \log_5^3) = t(3\log_5^2 - 2\log_5^3) \Rightarrow -\left(\frac{10}{24}+\frac{10}{14}\right) = t\left(3\times\frac{10}{24} + 2\times\frac{10}{14}\right) =$

$-\left(\frac{5}{12}+\frac{5}{7}\right) = t\left(\frac{5}{4}-\frac{10}{7}\right) \Rightarrow \frac{-95}{84} = t\left(-\frac{5}{28}\right)$

$\log_5^2 = \frac{1}{\log_2^5} = \frac{10}{24}$

$\log_5^3 = \frac{1}{\log_3^5} = \frac{10}{14}$

$\frac{-95}{84} \times \frac{-28}{5} = \frac{19}{3} = t \Rightarrow \frac{19}{3} \times 60 = 380$

(5)

$\frac{100}{100} - \frac{12.5}{100} = \frac{87.5}{100} = \frac{7}{8} \Rightarrow \frac{1}{7}m = m\left(\frac{7}{8}\right)^t \Rightarrow \log_2^{\frac{1}{7}} = t\log_2^{\frac{7}{8}} \Rightarrow -\log_2^7 = t(\log_2^7 - 3\log_2^2)$

$\frac{10}{4} \quad \frac{10}{4} \quad \frac{10}{12}$

$-\frac{10}{4} = t\left(\frac{10}{4} - \frac{30}{12}\right) \Rightarrow t = \frac{-10}{4} \times \frac{48}{-10} \Rightarrow t = 8 \Rightarrow 8 \times 7 = 56$ روز

$\frac{-20}{48} = \frac{-10}{24}$

(9)

$100\left(1-\frac{4}{100}\right)^n = \frac{1}{3}\times 100 \Rightarrow \left(\frac{96}{100}\right)^n = \frac{1}{3} \Rightarrow \left(\frac{24}{25}\right)^n = \frac{1}{3} \Rightarrow \log_{10}\left(\frac{24}{25}\right)^n = \log_{10}^{\frac{1}{3}} = n\log^{\frac{24}{25}} = -\log^3$

$\frac{48}{100}$

$n(\log^{24} - \log^{25}) \Rightarrow n(\log^3 + 3\log^2 - 2\log\frac{5}{10}) \Rightarrow n\left(\frac{138}{100} - 2\left(\log\frac{10}{10} - \log\frac{2}{10}\right)\right) \Rightarrow$

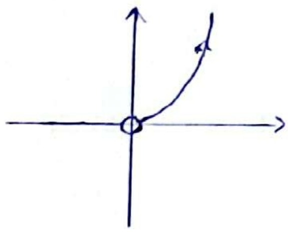
$\frac{48}{100} \quad \frac{90}{100} \qquad 1 \qquad \frac{7}{10} \quad \frac{3}{10}$

$\frac{-48}{100} = n\,\frac{-2}{100} \Rightarrow \frac{-48}{-2} = n \Rightarrow \frac{48}{2} = n \Rightarrow n = 24$

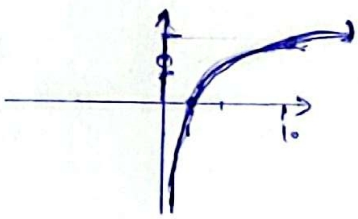
(10)

الف) $y = 9\log_3^x \Rightarrow x^{\log_3^9}$

$x > 0$



ب) $y = \log x^2 \Rightarrow 2\log x$



$x = 1 \Rightarrow y = 2$

$x > 0$

$x = 10 \Rightarrow 2$