


ضلع‌ها را a و b فرض کنیم:

$$\frac{a}{b} = \frac{3}{4} \quad S = 2 \times \left(\frac{1}{2} \times a \times b \times \sin 120^\circ \right) = \frac{ab}{2} = 54 \Rightarrow ab = 108$$



$$b = \frac{3}{4}a \rightarrow \frac{3}{4}a^2 = 108 \Rightarrow a = \sqrt{144} = 12 \rightarrow b = \frac{3}{4} \times 12 = 9$$

$$P = 2 \times (4\sqrt{3} + 9\sqrt{3}) = 26\sqrt{3}$$

$$S_{ABC} = \frac{1}{2} \times 4 \times 9 \times \sin A = 18 \sin A, \quad S_{ADE} = \frac{1}{2} \times 1 \times 1 \times \sin A = \frac{1}{2} \sin A$$

$$\Rightarrow \frac{S_{ABC}}{18 \sin A} - \frac{S_{ADE}}{\frac{1}{2} \sin A} = 1, 1 \Rightarrow 3 \sin A = 1, 1 \Rightarrow \sin A = \frac{1}{3} \Rightarrow \hat{A} = 30^\circ$$

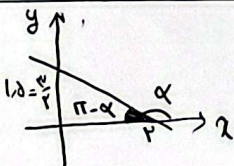
$$\tan \hat{A} = \tan 30^\circ = \frac{1}{\sqrt{3}} \quad \left(\frac{\sqrt{3}}{3} \right)$$

$$\frac{1}{\sqrt{\cos \alpha}} - \tan \alpha = \frac{1 + \sin \alpha}{|\cos \alpha|} \Rightarrow \frac{1}{|\cos \alpha|} - \tan \alpha = \frac{1 + \sin \alpha}{|\cos \alpha|} \Rightarrow \frac{1}{|\cos \alpha|} - \tan \alpha = \frac{1}{|\cos \alpha|} + \frac{\sin \alpha}{|\cos \alpha|}$$

$$\Rightarrow \frac{-\sin \alpha}{\cos \alpha} = \frac{\sin \alpha}{|\cos \alpha|} \Rightarrow |\cos \alpha| = -\cos \alpha \Rightarrow \cos \alpha < 0$$

$$\frac{|\sin \alpha|}{\cos \alpha} = -\frac{\sin \alpha}{\cos \alpha} \Rightarrow |\sin \alpha| = -\sin \alpha \Rightarrow \sin \alpha < 0$$

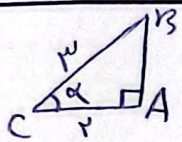
\Rightarrow $\sin \alpha$ و $\cos \alpha$ هر دو منفی هستند
پس انتخابی که α در ناحیه سوم
مربط به تکرار دارد



$$\tan(\pi - \alpha) = \frac{y}{x} = \frac{y}{-x} = -\frac{y}{x} = -\tan \alpha \quad \tan\left(\frac{\pi}{2} - \alpha\right) = \cot \alpha$$

$$\Rightarrow -\tan \alpha = \frac{y}{x} \Rightarrow \tan \alpha = -\frac{y}{x} \Rightarrow \frac{1}{\tan \alpha} = -\frac{x}{y} \Rightarrow \cot \alpha = \frac{x}{y} = \tan\left(\frac{\pi}{2} - \alpha\right)$$

$$\frac{3 \cos(120^\circ - 22^\circ) - 2 \sin(120^\circ - 22^\circ)}{\sin(120^\circ + 22^\circ) - \cos(120^\circ + 22^\circ)} = \frac{-3 \sin 22^\circ - 2 \sin 22^\circ}{-\sin 22^\circ - \sin 22^\circ} = \frac{-5 \sin 22^\circ}{-2 \sin 22^\circ} = \frac{5}{2}$$



$$\text{using } AB = \sqrt{a} \quad \left\{ \begin{array}{l} \sin \alpha = \frac{\sqrt{a}}{r} \\ \tan \alpha = \frac{\sqrt{a}}{r} \end{array} \right.$$

$$\frac{\sin(\alpha + \frac{\pi}{4}) - \sin(\alpha - \pi)}{|\tan \alpha - 1|} = \frac{\cos \alpha + \sin \alpha}{|\frac{a}{r} - 1|} = \frac{\frac{r}{r} - \frac{\sqrt{a}}{r}}{\frac{1}{r}} = \frac{r(1 - \sqrt{a})}{r}$$

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$$\sin \alpha = r \cos \alpha \xrightarrow{\div \cos \alpha} \tan \alpha = r \frac{1}{\cos \alpha} = 1 + \tan \alpha \rightarrow \frac{1}{\cos \alpha} = 2 \Rightarrow \cos \alpha = \pm \frac{\sqrt{a}}{2} \xrightarrow{r \cos \alpha} \cos \alpha = \frac{\sqrt{a}}{2}$$

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$$\tan \alpha = \tan 45^\circ = \sqrt{3} \quad m_1 + (m_1 - 1)y = r \Rightarrow \frac{0}{m_1 - 1} = \frac{-r}{\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \sqrt{3} m_1 + r m_1 - \sqrt{3} = 0 \Rightarrow |m_1 - m_2| = \frac{\sqrt{a}}{|a|} = \frac{\sqrt{4}}{\sqrt{3}} = \frac{r}{\sqrt{3}} = \frac{\sqrt{3}r}{3}$$

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$$-\frac{\pi}{4} < x < \frac{\pi}{4} \Rightarrow -\frac{\pi}{4} < -x < \frac{\pi}{4} \Rightarrow 0 < \frac{\pi}{4} - x < \frac{\pi}{4} \Rightarrow 0 < \tan(\frac{\pi}{4} - x)$$

$$\Rightarrow 0 < \frac{1-m}{r+m} \Rightarrow -r < m < 1$$

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$$\tan(45^\circ - 45^\circ) \cos(180^\circ + 45^\circ) + \tan(45^\circ + 180^\circ - 45^\circ) \sin(2 \times 180^\circ - 45^\circ)$$

$$= -\tan 45^\circ \times (-\cos 45^\circ) + (-\tan 45^\circ \times \sin 45^\circ) = (-\sqrt{2} \times \frac{\sqrt{2}}{2}) + (-\sqrt{2} \times \frac{\sqrt{2}}{2}) = \frac{r}{r} - \frac{r}{r} = 0$$

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