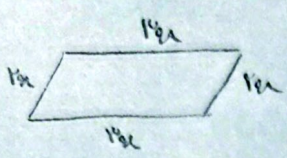


نام و نام خانوادگی ..... پانصد و پنجاه و یک شماره کلاس ... کلاس ... دبیرستان ...

در یک مثلث ارتفاع به مساحت ۳۴ و نسبت دو ضلع برابر ۳ به ۲ است. اگر زاویه بدلتین این ضلع ۲ برابر آن باشد. مساحت این مثلث را بیابید.



$$S = \frac{1}{2} \times a \times h = 34$$

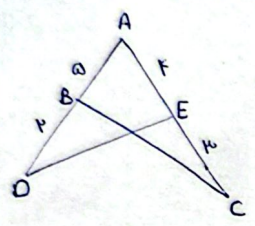
$$\frac{1}{2} \times x \times h + \frac{1}{2} \times (a-x) \times h = 34 \Rightarrow \frac{1}{2} \times a \times h = 34 \Rightarrow ah = 68$$

$$\frac{h}{a} = \frac{3}{2} \Rightarrow h = \frac{3}{2}a \Rightarrow \frac{3}{2}a \times a = 68 \Rightarrow \frac{3}{2}a^2 = 68 \Rightarrow a^2 = \frac{136}{3} \Rightarrow a = \sqrt{\frac{136}{3}}$$

$$h = \frac{3}{2} \times \sqrt{\frac{136}{3}} = \frac{3}{2} \times \frac{\sqrt{136}}{\sqrt{3}} = \frac{3\sqrt{136}}{2\sqrt{3}} = \frac{3\sqrt{4 \times 34}}{2\sqrt{3}} = \frac{6\sqrt{34}}{2\sqrt{3}} = \frac{3\sqrt{34}}{\sqrt{3}} = \sqrt{34} \times \sqrt{3} = \sqrt{102}$$

$$S = \frac{1}{2} \times a \times h = \frac{1}{2} \times \sqrt{\frac{136}{3}} \times \sqrt{102} = \frac{1}{2} \times \sqrt{\frac{136 \times 102}{3}} = \frac{1}{2} \times \sqrt{\frac{136 \times 34 \times 3}{3}} = \frac{1}{2} \times \sqrt{136 \times 34} = \frac{1}{2} \times \sqrt{4624} = \frac{1}{2} \times 68 = 34$$

در مثلث زیر ارتفاع دو مساحت های مختلف ABC و ADE برابر ۱/۳ است. tan A را بیابید. (A زاویه است)



$$S_{ABC} - S_{ADE} = 1/3 \quad (I)$$

$$\left. \begin{aligned} S_{ABC} &= \frac{1}{2} \times (f + 3) \times \sin A = \frac{1}{2} \times 4 \times \sin A = 2 \sin A \\ S_{ADE} &= \frac{1}{2} \times (1 + 2) \times \sin A = \frac{1}{2} \times 3 \times \sin A = \frac{3}{2} \sin A \end{aligned} \right\} \Rightarrow 2 \sin A - \frac{3}{2} \sin A = \frac{1}{3} \Rightarrow \frac{1}{2} \sin A = \frac{1}{3} \Rightarrow \sin A = \frac{2}{3}$$

$$(I) \Rightarrow \frac{1}{2} \sin A = \frac{1}{3} \Rightarrow \sin A = \frac{2}{3}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{2/3}{\sqrt{1 - 4/9}} = \frac{2/3}{\sqrt{5/9}} = \frac{2/3}{\sqrt{5}/3} = \frac{2}{\sqrt{5}}$$

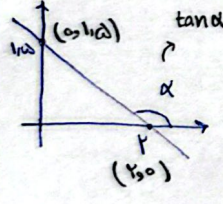
اگر  $\frac{1}{|\cos \alpha|} - \frac{\sin \alpha}{\cos \alpha} = \frac{1 + \sin \alpha}{|\cos \alpha|}$  باشد.  $\frac{1}{\sqrt{\cos^2 \alpha}} - \tan \alpha = \frac{1 + \sin \alpha}{|\cos \alpha|}$  را بیابید.

$$\frac{1}{|\cos \alpha|} - \frac{\sin \alpha}{\cos \alpha} = \frac{1 + \sin \alpha}{|\cos \alpha|} \Rightarrow \frac{1 - \sin \alpha}{|\cos \alpha|} = \frac{1 + \sin \alpha}{|\cos \alpha|} \Rightarrow 1 - \sin \alpha = 1 + \sin \alpha \Rightarrow -2 \sin \alpha = 0 \Rightarrow \sin \alpha = 0$$

$$\frac{|\sin \alpha|}{\cos \alpha} = \frac{-\sin \alpha}{\cos \alpha} \Rightarrow \sin \alpha < 0$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{-\sin \alpha}{\cos \alpha} = -\frac{|\sin \alpha|}{\cos \alpha}$$

در مثلثی که زاویه  $\alpha$  ضلعین شده است. مثلث  $(\frac{\pi}{2} - \alpha)$  را بیابید.



$$\tan \alpha = \frac{b}{c} = m \Rightarrow y - b = m(x - a)$$

$$\left. \begin{aligned} (0, 1/a) \rightarrow y - 1/a = m(x - a) \Rightarrow y = mx + 1/a \\ (x_0, 0) \rightarrow y - 0 = m(x - x_0) \Rightarrow y = mx - mx_0 \end{aligned} \right\} \Rightarrow mx + 1/a = mx - mx_0 \Rightarrow 1/a = -mx_0 \Rightarrow m = -1/(ax_0)$$

$$\tan(\frac{\pi}{2} - \alpha) = \frac{\tan \frac{\pi}{2} - \tan \alpha}{1 + \tan \frac{\pi}{2} \cdot \tan \alpha} = \frac{1 - 0/a}{1 + (1 \times 0/a)} = \frac{1}{1} = 1$$

$$\tan(\frac{\pi}{2} - \alpha) = \cot \alpha = \frac{1}{\tan \alpha} = \frac{1}{m}$$

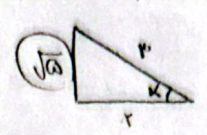
مطلوبه عبارت  $\frac{3 \cos(2\pi/3) - 2 \sin(2\pi/3)}{\sin(\pi/2) - \cos(\pi/2)}$  را بیابید.

$$\frac{3 \cos(2\pi/3) - 2 \sin(2\pi/3)}{\sin(\pi/2) - \cos(\pi/2)} = \frac{3 \cos(120^\circ) - 2 \sin(120^\circ)}{\sin(90^\circ) - \cos(90^\circ)} = \frac{3 \cos(120^\circ) - 2 \cos(120^\circ)}{\cos(-120^\circ) - (-\cos(120^\circ))}$$

$$= \frac{3 \cos(120^\circ) - 2 \cos(120^\circ)}{\cos(120^\circ) + \cos(120^\circ)} = \frac{\cos(120^\circ)}{2 \cos(120^\circ)} = \frac{1}{2}$$

$$\frac{-3 \sin 2\pi/3 - 2 \sin 2\pi/3}{-\sin 2\pi/3 - \sin 2\pi/3} = \frac{-5 \sin 2\pi/3}{-2 \sin 2\pi/3} = \frac{5}{2}$$

$\frac{\sin(\frac{\pi}{r} + \alpha) - \sin(\alpha - \pi)}{|\tan^r \alpha - 1|}$



$r + \alpha = \pi$   
 $\sin \alpha = \frac{\sqrt{a}}{r}$   
 $\tan \alpha = \frac{\sqrt{a}}{r}$

$\frac{\cos(\alpha) - \sin(\alpha)}{|\tan^r \alpha - 1|} = \frac{r - \sqrt{a}}{r} = \frac{r - r\sqrt{a}}{r}$

$\Rightarrow \frac{r - \sqrt{a}}{r} = \frac{1 - \sqrt{a}}{1}$

$\sin \alpha = r \cos \alpha \rightarrow \sin^2 \alpha = r^2 \cos^2 \alpha \quad (I) \rightarrow 1 - \cos^2 \alpha = r^2 \cos^2 \alpha$   
 $1 = (r^2 + 1) \cos^2 \alpha$   
 $\frac{1}{r^2 + 1} = \cos^2 \alpha$

$\sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow 1 - \cos^2 \alpha = \sin^2 \alpha \quad (II)$

$\left(\frac{1}{\sqrt{r^2 + 1}}\right) = \cos \alpha \rightarrow \frac{1}{\sqrt{a}} = \cos \alpha$

خط  $r \cos \alpha + (m^2 - 1) y = r$  از این دو معادله  $m$  به دست می آید.

$\tan \alpha = \sqrt{r}$

$r \cos \alpha + (m^2 - 1) y = r \rightarrow y = \frac{-r \cos \alpha}{m^2 - 1} + \frac{r}{m^2 - 1}$

$\Delta = 14 \quad \frac{-r \pm r}{r \sqrt{r}}$

$\frac{-r}{m^2 - 1} = \sqrt{r} \rightarrow \sqrt{r} m^2 - \sqrt{r} = -r \rightarrow \sqrt{r} m^2 + r - \sqrt{r} = 0$

$m_1 = \frac{1}{\sqrt{r}} \quad m_2 = \frac{-r}{\sqrt{r}}$

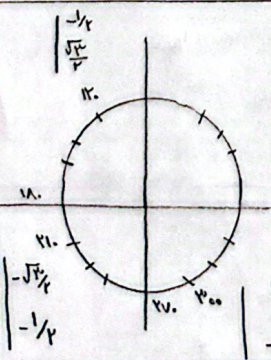
اختلاف  $m$ :  $\frac{1}{\sqrt{r}} - \left(-\frac{r}{\sqrt{r}}\right) = \frac{r}{\sqrt{r}}$

$\frac{\pi}{r} - \alpha$ : (ربع اول)  $\rightarrow$

$\tan\left(\frac{\pi}{r} - \alpha\right) = \frac{1 - m}{r + m}$  و  $\frac{\pi}{r} < \alpha < \frac{\pi}{r}$

$-\frac{\pi}{r} < \alpha < \frac{\pi}{r} \Rightarrow 0 < \frac{\pi}{r} - \alpha < \frac{\pi}{r}$

مقدار نزول و ربع اول:  $\frac{1 - m}{r + m} > 0 \Rightarrow \frac{-r}{-\phi + \phi} \Rightarrow -r < m < 1$



$\tan(\alpha_0) \cos(\alpha_1) + \tan(\alpha_1) \sin(\alpha_0)$

$(-\sqrt{r} \times \frac{\sqrt{r}}{r}) + (\sqrt{r} \times \frac{\sqrt{r}}{r})$   
 $(-\frac{r}{r}) + (\frac{r}{r}) = 0$

$\alpha_1 = \alpha_0 + \pi$   
 $\alpha_0 = r \times \alpha_1 + (\pi)$

$\tan(\pi) = \sqrt{r}$   
 $\sin(\pi) = \frac{\sqrt{r}}{r}$