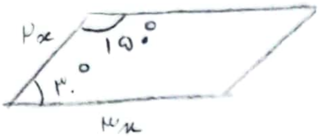
۱



$S_{\text{متوازی الاضلاع}} = 2\times S_{\triangle} = 2\times \frac{1}{2}\times 2x\times 3x\times \underbrace{\sin 30°}_{\frac{1}{2}} = 54$

$3x^2 = 54 \to x^2 = 18 \to x = 3\sqrt{2}$

$\text{محیط} = 2(\underbrace{2x+3x}_{5x}) = \boxed{30\sqrt{2}}$

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$A\hat{B}C$ $\text{مساحت} = \frac{1}{2}\times AB\times BC\times \sin\hat{B} = \frac{1}{2}AC\times BC\times \sin\hat{C} = \frac{1}{2}AC\times AB\times \sin\hat{A}$

مساحت هر دو مثلث را برحسب سینوس زاویه A می‌نویسیم و طبق فرض داریم:

$S_{A\hat{B}C} - S_{A\hat{D}E} = 1.75 \Rightarrow \frac{1}{2}AB\times AC\times \sin\hat{A} - \frac{1}{2}AD\times AE\times \sin\hat{A} = 1+\frac{3}{4}$

$\Rightarrow \frac{1}{2}\times 5\times 7\times \sin\hat{A} - \frac{1}{2}\times 7\times 4\times \sin\hat{A} = \frac{7}{4} \Rightarrow 35\sin\hat{A} - 28\sin\hat{A} = \frac{7}{2} \Rightarrow 7\sin\hat{A} = \frac{7}{2}$

$\sin\hat{A} = \frac{1}{2}$

با توجه به اینکه زاویه $\hat{A}$ حاده است و سینوس A برابر $\frac{1}{2}$ پس $\hat{A}=30°$ و مقدار خواسته شده برابر است با:

$\boxed{\tan\hat{A} = \tan 30° = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}}$

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$\frac{|\sin a|}{\cos a} = \frac{-1}{\cot a}$ ، $\frac{1}{\sqrt{\cos^2 a}} - \tan a = \frac{1+\sin a}{|\cos a|}$

$\frac{1}{|\cos a|} - \frac{\sin a}{\cos a} = \frac{1+\sin a}{|\cos a|} \longrightarrow \cos a < 0$

$\Longrightarrow$ $\boxed{\text{ناحیه سوم}}$

$\frac{|\sin a|}{\cos a} = \frac{-\sin a}{\cos a} \longrightarrow \sin a < 0$

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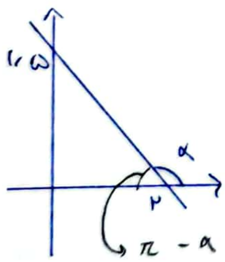
$\underbrace{\tan\left(\frac{\pi}{2} - \alpha\right)}_{\cot\alpha}$



$\tan(\pi - \alpha) = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \tan\alpha = -\frac{3}{4} \Rightarrow \boxed{\cot\alpha = -\frac{4}{3}}$

۵

$\frac{3\cos(248°) - 2\sin(158°)}{\sin(202°) - \cos(292°)} = ?$

$\frac{3\cos\left(\frac{3\pi}{2} - 22°\right) - 2\sin(\pi - 22°)}{\sin(\pi + 22°) - \cos\left(\frac{3\pi}{2} + 22°\right)}$

$= \frac{-3\sin(22) - 2\sin(22)}{-\sin(22) - \sin(22)} = \frac{-5}{-2} = \boxed{2.5}$

$$\frac{\sin(\frac{\pi}{2}+\alpha)-\sin(\alpha-\pi)}{|\tan^2\alpha-1|} \overset{\star}{=} \qquad \cos\alpha=\frac{2}{3} \quad \text{ربع چهارم}$$ ۶

$$\sin^2\alpha+\frac{4}{9}=1 \rightarrow \sin\alpha=-\frac{\sqrt{5}}{3}$$

$$\frac{\sin^2\alpha}{\cos^2\alpha}-1=\left|\frac{\sin^2\alpha-1}{\cos^2\alpha}\right|=\left|\frac{\sin^2\alpha-\cos^2\alpha}{\cos^2\alpha}\right|=\left|\frac{1-2\cos^2\alpha}{\cos^2\alpha}\right|=\frac{1-\frac{8}{9}}{\frac{4}{9}}=\frac{\frac{1}{9}}{\frac{4}{9}}=\frac{1}{4}$$

$$\overset{\star}{=}\ \frac{\cos\alpha+\sin\alpha}{1/4}=\frac{\frac{2-\sqrt{5}}{3}}{1/4}=\frac{8-4\sqrt{5}}{3}$$

$$\sin\alpha=2\cos\alpha \longrightarrow \frac{\sin\alpha}{\cos\alpha}=2 \longrightarrow \tan\alpha=2 \qquad \text{ربع سوم}$$ ۷

$$1+\tan^2\alpha=\frac{1}{\cos^2\alpha} \rightarrow 1+4=\frac{1}{\cos^2\alpha} \rightarrow \cos^2\alpha=\frac{1}{5} \rightarrow \cos\alpha=\pm\frac{1}{\sqrt{5}} \xrightarrow{\text{ربع سوم}} \cos\alpha=-\frac{1}{\sqrt{5}}=-\frac{\sqrt{5}}{5}$$

$$2mx+(m^2-1)y=3 \qquad \tan 60^\circ=\sqrt{3} \rightarrow \frac{-2m}{m^2-1}=\sqrt{3}$$ ۸

$$\sqrt{3}m^2-\sqrt{3}=-2m \rightarrow \sqrt{3}m^2+2m-\sqrt{3}=0 \rightarrow \Delta=(2)^2-4(\sqrt{3})(-\sqrt{3})=4+12=16>0$$

$$\begin{cases} m_1=\frac{-2+4}{2\sqrt{3}}=\frac{2}{2\sqrt{3}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3} \\ m_2=\frac{-2-4}{2\sqrt{3}}=\frac{-6}{2\sqrt{3}}=\frac{-3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{-3\sqrt{3}}{3} \end{cases} \longrightarrow |m_1-m_2|=\left|\frac{\sqrt{3}}{3}+\frac{3\sqrt{3}}{3}\right|=\frac{4\sqrt{3}}{3}$$

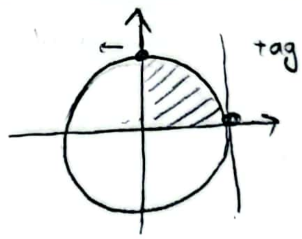
$$\tan(\frac{\pi}{4}-x)=\frac{1-m}{2+m}, \quad -\frac{\pi}{4}<x<\frac{\pi}{2}$$ ۹

(1) $-\frac{\pi}{4}<-x<\frac{\pi}{4} \rightarrow 0<\frac{\pi}{4}-x<\frac{\pi}{2}$

(2) $\frac{1-m}{2+m}>0 \longrightarrow$

-2		1	
-	+		-

$(-2, 1)$



$$\tan(300^\circ)\cos(210^\circ)+\tan(480^\circ)\sin(840^\circ)= \longrightarrow \left(-\sqrt{3}\times-\frac{\sqrt{3}}{2}\right)+\left(-\sqrt{3}\times\frac{\sqrt{3}}{2}\right)=\frac{3}{2}-\frac{3}{2}=\boxed{0}$$ ۱۰

$$\tan(300)=\tan(360-60)=\tan(-60)=-\sqrt{3}$$

$$\cos(210)=\cos(\underbrace{180+30}_{\text{ربع سوم}})=-\cos 30=-\frac{\sqrt{3}}{2}$$

$$\tan(480)=\tan 120=\tan(\underbrace{180-60}_{\text{ربع دوم}})=-\tan 60=-\sqrt{3}$$

$$\sin(840)=\sin 120=\sin(\underbrace{180-60}_{\text{ربع دوم}})=\sin 60=\frac{\sqrt{3}}{2}$$