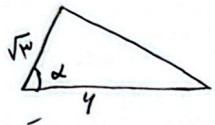


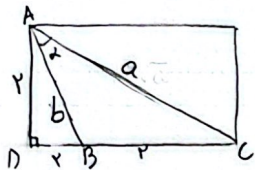
$$S = \frac{1}{2} \times 4 \times \sqrt{3} \times \sin \alpha = 4 \Rightarrow \sin \alpha = \frac{4}{4\sqrt{3}} = \frac{\sqrt{3}}{3}$$



$$\sin \alpha = \frac{\sqrt{3}}{3} \Rightarrow \alpha_1 = \frac{\pi}{3}, \alpha_2 = \frac{2\pi}{3} \Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{\frac{\pi}{3}}{\frac{2\pi}{3}} = \frac{1}{2}$$

۱

$$b = 2\sqrt{2}, a = 2\sqrt{5} \quad \triangle ADC \text{ متساوی الساقین } \frac{1}{2} \times 2 \times 2 \times \sin 90^\circ = 2$$



$$S_{ADB} = \frac{1}{2} \times 2 \times 2 \times \sin 90^\circ = 2 \Rightarrow S_{ABC} = S_{ADC} - S_{ADB} = 2$$

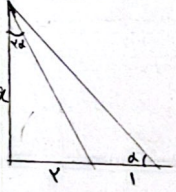
$$S_{ABC} = \frac{1}{2} \times 2 \times \sqrt{2} \times \sqrt{5} \times \sin \alpha = 2 \Rightarrow \sin \alpha = \frac{\sqrt{10}}{10}$$

$$\cos^2 \alpha = 1 - \sin^2 \alpha = 1 - \frac{10}{100} = \frac{90}{100} \Rightarrow \cos \alpha = \frac{3\sqrt{10}}{10}$$

$$\cot \alpha = \frac{3\sqrt{10}}{\sqrt{10}} = 3$$

۲

$$\tan^2 \alpha = \frac{y}{x} \quad \text{و} \quad \tan \alpha = \frac{x}{y}$$



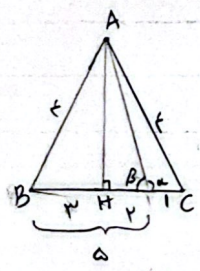
$$\tan^2 \alpha = \frac{y \tan \alpha}{1 - \tan^2 \alpha} \rightarrow \frac{y}{x} = \frac{\frac{y}{x}}{1 - \frac{y^2}{x^2}} \Rightarrow \frac{y}{x} = \frac{y}{x - \frac{y^2}{x}}$$

$$1 - \frac{y^2}{x^2} = \frac{y^2}{x^2} \rightarrow 1 - 2\frac{y^2}{x^2} = 0 \Rightarrow \frac{y^2}{x^2} = \frac{1}{2} \Rightarrow \frac{y}{x} = \frac{1}{\sqrt{2}}$$

$$\tan \alpha = \frac{1/\sqrt{2}}{1} = \frac{1}{\sqrt{2}} \Rightarrow \cot \alpha = \sqrt{2}$$

۳

$$AH^2 = (AC)^2 - (CH)^2 = 14 - 9 = 5 \Rightarrow AH = \sqrt{5}$$



$$\tan \alpha = \frac{AH}{BH} = \frac{\sqrt{5}}{y}$$

۴

$$y \sin^2 \alpha + \cos^2 \alpha = \frac{5}{4}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1 \Rightarrow \sin^2 \alpha = 1 - \cos^2 \alpha \Rightarrow y - y \cos^2 \alpha + \cos^2 \alpha = \frac{5}{4}$$

$$\Rightarrow -\cos^2 \alpha = \frac{5}{4} - y \Rightarrow \cos^2 \alpha = \frac{y}{4} \quad , \quad 1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha} \Rightarrow 1 + \tan^2 \alpha = \frac{4}{y}$$

$$\tan^2 \alpha = \frac{4}{y} - \frac{y}{4} = \frac{1}{4}$$

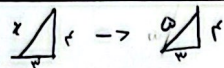
۵

$$\frac{\sin^2 \alpha + t \cos^2 \alpha}{1 + \cos^2 \alpha} - \frac{\cos^2 \alpha + t \sin^2 \alpha}{1 + \sin^2 \alpha} = \frac{\sin^2 \alpha + t(1 - \sin^2 \alpha)}{1 + \cos^2 \alpha} - \frac{\cos^2 \alpha + t(1 - \cos^2 \alpha)}{1 + (1 - \cos^2 \alpha)}$$

$$= \frac{\sin^2 \alpha - t \sin^2 \alpha + t}{1 + \sin^2 \alpha} - \frac{\cos^2 \alpha - t \cos^2 \alpha + t}{1 + \sin^2 \alpha} = \frac{(1 - \sin^2 \alpha)^t}{1 + \sin^2 \alpha} - \frac{(1 - \cos^2 \alpha)^t}{1 + \sin^2 \alpha}$$

$$1 - \sin^2 \alpha - 1 - \cos^2 \alpha = \cos^2 \alpha - \sin^2 \alpha = \cos^2 \alpha$$

6

$\tan \alpha = \frac{f}{v}$ ,  $\sin \alpha, \cos \alpha < 0$    $\cos \alpha = -\frac{v}{r}$ ,  $\sin \alpha = -\frac{f}{r}$ ,  $\cot \alpha = \frac{v}{f}$

$$\sin\left(\frac{9\pi}{v} + \alpha\right) \cos\left(\frac{v\pi}{v} - \alpha\right) - \frac{\tan\left(\alpha - \frac{v\pi}{v}\right)}{-\cot \alpha} = \sin\left(\frac{\pi}{v} + \alpha\right) \cos\left(\frac{\pi}{v} - \alpha\right) + \cot \alpha$$

$$\Rightarrow \cos \alpha (-\sin \alpha) + \cot \alpha = -\frac{v}{r} \times \left(-\frac{f}{r}\right) + \frac{v}{f} = \frac{fv}{r^2}$$

7

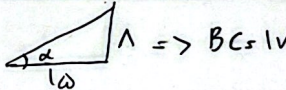
$x = \frac{\pi}{14}$   $\sin \frac{\pi}{14} < \cos \frac{\pi}{14}$

$$v \cos^2 \alpha + \sqrt{v} \sin \alpha - \sqrt{v} \cos \alpha = \frac{v}{v} + \sqrt{v} \left( \sin \frac{\pi}{14} - \cos \frac{\pi}{14} \right)$$

$$\left( \sin \frac{\pi}{14} - \cos \frac{\pi}{14} \right)^2 \sin^2 \frac{\pi}{14} + \cos^2 \frac{\pi}{14} - v \sin \frac{\pi}{14} \cos \frac{\pi}{14} = 1 - \sin \frac{\pi}{4} = \frac{1}{v} \Rightarrow \sin \frac{\pi}{14} - \cos \frac{\pi}{14} = \frac{-1}{\sqrt{v}}$$

$$\Rightarrow \frac{v}{v} + \sqrt{v} \left( \frac{-1}{\sqrt{v}} \right) = \frac{v}{v} - 1 = \frac{1}{v}$$

8

$\tan\left(\frac{\alpha}{v}\right) = \frac{1}{f}$   $\tan \alpha = \frac{v \tan \frac{\alpha}{v}}{1 - \tan^2 \frac{\alpha}{v}} = \frac{v \left(\frac{1}{f}\right)}{1 - \frac{1}{f^2}} = \frac{1}{10}$  

$\sin \alpha = \frac{1}{14}$ ,  $\cos \alpha = \frac{13}{14}$

$$\frac{\tan \alpha - \sin \alpha}{\sin \alpha - \cos \alpha} = \frac{\frac{1}{10} - \frac{1}{14}}{\frac{1}{14} - \frac{13}{14}} = \frac{-14}{100}$$

9

$v \sin \alpha < \sin^2 \alpha$ ,  $\frac{\cot \alpha}{\sin \alpha} > 0$

$\sin^2 \alpha = v \sin \alpha \cos \alpha \rightarrow v \sin \alpha < v \sin \alpha \cos \alpha \rightarrow v \sin \alpha (1 - \cos \alpha) < 0 \Rightarrow \sin \alpha < 0$

$\sin \alpha$  منفی است و  $\frac{\cot \alpha}{\sin \alpha}$  مثبت در نتیجه  $\cot \alpha$  هم باید منفی باشد و این در صورتی است که  $\sin \alpha$  و  $\cos \alpha$  هم علامت داشته باشند در نتیجه  $\cos \alpha$  مثبت است و  $\alpha$  در ربع چهارم است.

$\sin \alpha < 0$ ,  $\cos \alpha > 0 \Rightarrow$  ربع چهارم

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