(۷

$\sin\alpha < 0 \qquad \cos\alpha < 0 \qquad + \tan(\frac{\pi}{۲} - \alpha)$

$\sin(\frac{\pi}{۲}+\alpha)\cos(\frac{\pi}{۲}-\alpha) - \cancel{\tan(\alpha - \frac{۳\pi}{۲})}$

$(\cos\alpha)(\sin\alpha) + \cot\alpha = \sin\alpha\cos\alpha + \frac{\cos\alpha}{\sin\alpha} = \frac{\sin^۲\alpha\cos\alpha + \cos\alpha}{\sin\alpha} = \frac{\cos\alpha(\sin^۲+۱)}{\sin^\alpha}$

$\frac{\cos\alpha((۱-\cos^۲\alpha)+۱)}{\sin\alpha} = \frac{\cos\alpha(\cos^۲\alpha)}{\sin\alpha} = \frac{\cos^۳\alpha}{\sin\alpha} \qquad \tan\alpha = \frac{۴}{۳} = \frac{\sin\alpha}{\cos\alpha}$

$\to \frac{(۳)^۳}{۴} = \frac{-۲۷}{-۴}$ (۶,۷۵)

(۸

$(۲\cos\frac{\pi}{۳} + \sqrt{۲}\sin\frac{\pi}{۱۲} - \sqrt{۲}\cos\frac{\pi}{۱۲})$

$(\underbrace{۲\cos ۶۰^\circ}_{\frac{۳}{۲}} + \sqrt{۲}\sin ۱۵^\circ - \sqrt{۲}\cos ۱۵^\circ)$

$\frac{۳}{۲} + \sqrt{۲}(\sin(\frac{\pi}{۳}-\frac{\pi}{۴}) - \cos(\frac{\pi}{۳}-\frac{\pi}{۴})) \to \frac{۳}{۲} + \sqrt{۲}(\frac{\sqrt{۳}-\sqrt{۲}}{۲}) - (\frac{۱-\sqrt{۲}}{۲})$

$\frac{۳}{۲} + \sqrt{۲}(\frac{\sqrt{۳}-\sqrt{۶}-۱+\sqrt{۲}}{۲}) = \frac{۳+\sqrt{۶}-\sqrt{۲}}{۲}$

(۹

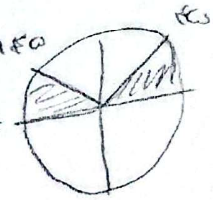
$\boxed{\tan(\frac{\alpha}{۲}) = \frac{۱}{۴}} \qquad \tan\frac{\alpha}{۲} = \frac{\sin\alpha}{۱+\cos\alpha} = \frac{۱-\cos\alpha}{\sin\alpha} \to ۴-۴\cos\alpha = \frac{۱+\cos\alpha}{۴}$

$\Rightarrow \cos\alpha = \frac{۱۵}{۱۷} \qquad \sin = \frac{۸}{۱۷}$

$\tan = \frac{۸}{۱۵}$

$\frac{\frac{۸}{۱۵} - \frac{۸}{۱۷}}{\frac{۸}{۱۷} - \frac{۱۵}{۱۷}} = \frac{\frac{۱۶}{\cancel{۱۷}\times۱۵}}{\frac{-۷}{\cancel{۱۷}}} = \frac{\frac{۱۶}{۱۵}}{\frac{-۷}{۱}} = \boxed{-\frac{۱۶}{۱۰۵}}$

(۱۰

$۲\sin\alpha < \sin ۲\alpha \Rightarrow \alpha =$ 

$\Rightarrow ۰ < \alpha \le ۴۵ \cup ۱۳۵ \le \alpha < ۱۸۰$ ربع اول و دوم

$۰ < \frac{\cot\alpha}{\sin\alpha} \Rightarrow \frac{\frac{\cos\alpha}{\sin\alpha}}{\frac{\sin\alpha}{۱}} : ۰ < \frac{\cos\alpha}{\sin^۲\alpha} \Rightarrow \cos\alpha > ۰ \Rightarrow$ ربع اول و چهارم

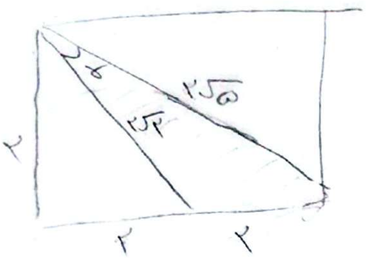
$\longrightarrow$ ربع اول $\to ۰ < \alpha \le ۴۵$

۱) $\frac{۱}{۲}\times ۶\times ۶\sqrt{۳}\times \sin\alpha = ۴۵ \Rightarrow \sin\alpha = \frac{\sqrt{۳}}{۲}$

$\sin\alpha = \frac{\sqrt{۳}}{۲}\begin{cases}\alpha = \frac{\pi}{۳}\\ \alpha = \frac{۲\pi}{۳}\end{cases}$

$\frac{\alpha_{max}}{\alpha_{min}} = \frac{\frac{۲\pi}{۳}}{\frac{\pi}{۳}} = ۲$

۲)



$S = ۴ - ۲ = ۲\sqrt{۵}\times \cancel{۲}\sqrt{۲}\times \frac{۱}{\cancel{۲}}\times \sin\alpha \Rightarrow$

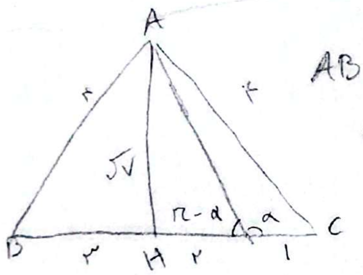
$۲\sqrt{۱۰}\sin\alpha = ۲ \quad \sin\alpha = \frac{\cancel{۲}}{\cancel{۲}\sqrt{۱۰}} = \frac{۱}{\sqrt{۱۰}} = \boxed{\frac{\sqrt{۱۰}}{۱۰}}$

$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\frac{\sqrt{۹۰}}{\cancel{۱۰}}}{\frac{\sqrt{۱۰}}{\cancel{۱۰}}} = \frac{\sqrt{۹۰}}{\sqrt{۱۰}} \Rightarrow \cos\alpha = \frac{\sqrt{۹۰}}{۱۰}$

$\rightarrow \frac{۳\cancel{\sqrt{۱۰}}}{\cancel{\sqrt{۱۰}}} = ۳$

۳)

۴)



$AB^۲ = AH^۲ + BH^۲ = ۱۶ = AH^۲ + ۹$

$\Rightarrow AH = \sqrt{۷}$

$\tan(\pi - \alpha) = \frac{\sqrt{۷}}{۲} \rightarrow -\tan\alpha = \frac{\sqrt{۷}}{۲} \rightarrow \tan\alpha = -\frac{\sqrt{۷}}{۲}$

۵)

$\sin^۲ x + \underbrace{\sin^۲ x + \cos^۲ x}_{۱} = \frac{۴}{۳} \Rightarrow \sin^۲ x = \frac{۱}{۳} \rightarrow \sin x = \frac{۱}{\sqrt{۳}} \Rightarrow \cos x = \frac{\sqrt{۲}}{\sqrt{۳}}$

$\tan^۲ x = \left(\frac{\frac{۱}{\cancel{\sqrt{۳}}}}{\frac{\sqrt{۲}}{\cancel{\sqrt{۳}}}}\right)^۲ = \left(\frac{۱}{\sqrt{۲}}\right)^۲ = \frac{۱}{۲}$

۶)

$\frac{\sin^۴\alpha + ۴(۱-\sin^۲\alpha)}{۱+(۱-\sin^۲\alpha)} - \frac{\cos^۴\alpha + ۴(۱-\cos^۲\alpha)}{۱+(۱-\cos^۲\alpha)} = \frac{\sin^۴\alpha + ۴ - ۴\sin^۲\alpha}{۲-\sin^۲\alpha} - \frac{\cos^۴\alpha + ۴ - ۴\cos^۲\alpha}{۲-\cos^۲\alpha}$

$\frac{(\sin^۲\alpha - ۲)^۲}{(۲-\sin^۲\alpha)} - \frac{(۲-\cos^۲\alpha)^۲}{۲-\cos^۲\alpha} = \sin^۲\alpha - ۲ - ۲ + \cos^۲\alpha = ۱ - ۴ = -۳$