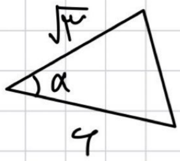
(۱۸) سازمان سامانی تألیف شماره ۲۷ _ یازدهم دفتر A

۱_



$S=\frac{1}{2}ab\sin\alpha \Rightarrow \frac{1}{2}\times\sqrt{3}\times 6\times\sin\alpha=4.5 \Rightarrow \sin\alpha=\frac{3}{2\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{3\sqrt{3}}{6}=\frac{\sqrt{3}}{2}$

$\alpha=60^\circ$ یا $120^\circ \Rightarrow \frac{120}{60}=2$

(۲)

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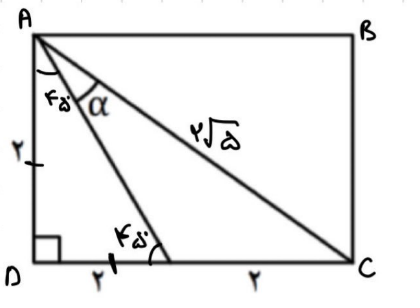


$AC^2=DC^2+AD^2 \Rightarrow AC=2\sqrt{5}$

$\cot(\alpha+\beta)=\frac{\cot\alpha\cot\beta-1}{\cot\beta+\cot\alpha} \Rightarrow \frac{\cot\alpha-1}{1+\cot\alpha}=\frac{1}{2} \Rightarrow$

$\cot(\alpha+45^\circ)=\frac{2}{4}=\frac{1}{2}$

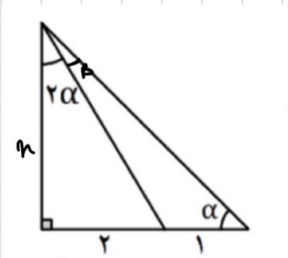
$\cot 45^\circ=1$

$2\cot\alpha-2=1+\cot\alpha \Rightarrow$

$\cot\alpha=3$

(۴)

۳_



$\cot\alpha=\frac{3}{x}$

$\cot 2\alpha=\frac{x}{2} \Rightarrow \cot(\alpha+\alpha)=\frac{\cot\alpha\cot\alpha-1}{\cot\alpha+\cot\alpha} \Rightarrow$

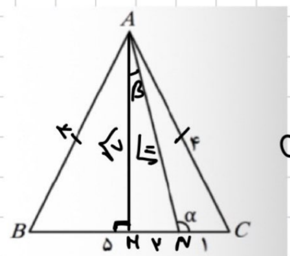
(۲)

$\frac{\frac{9}{x^2}-1}{\frac{6}{x}}=\frac{x}{2} \Rightarrow \frac{\frac{9-x^2}{x^2}}{\frac{6}{x}}=\frac{9-x^2}{6x}=\frac{x}{2} \Rightarrow 18-2x^2=6x^2$

$8x^2=18 \Rightarrow x^2=\frac{18}{8} \Rightarrow x=\frac{3\sqrt{2}}{2\sqrt{2}}=\frac{3}{2}$

$\cot\alpha=\frac{3}{x}=\frac{3}{\frac{3}{2}}=2$

۴_



$AB^2=BH^2+AH^2 \Rightarrow AH=\sqrt{7}$, $AN^2=NH^2+AH^2 \Rightarrow AN=\sqrt{11}$

زاویه خارجی $\alpha=\beta+90^\circ \Rightarrow \cos\alpha=-\sin\beta$, $\sin\alpha=\cos\beta \Rightarrow \cos\alpha=\frac{-2}{\sqrt{11}}$, $\sin\alpha=\frac{\sqrt{7}}{\sqrt{11}} \Rightarrow \tan\alpha=\frac{\frac{\sqrt{7}}{\sqrt{11}}}{\frac{-2}{\sqrt{11}}}=\frac{-\sqrt{7}}{2}$

(۴)

-۵

$$۲\sin^۲\theta + \cos^۲\theta = \frac{۶}{۳}$$

$$\sin^۲\theta + \cos^۲\theta = ۱$$

$$\sin^۲\theta = \frac{۱}{۳} \Rightarrow \cos^۲\theta = \frac{۲}{۳} \Rightarrow \tan^۲\theta = \frac{\sin^۲\theta}{\cos\theta} = \frac{\frac{۱}{۳}}{\frac{۲}{۴}} = \frac{۱}{۲}$$

۶

-۶

$$\frac{\sin^۴\alpha + ۴\cos^۲\alpha}{۱+\cos^۲\alpha} - \frac{\cos^۴\alpha + ۴\sin^۲\alpha}{۱+\sin^۲\alpha} =$$

$$\sin^۲\alpha = ۱ - \cos^۲\alpha$$

$$\cos^۲\alpha = ۱ - \sin^۲\alpha$$

۱,۵

$$\frac{(۱-\cos^۲\alpha)^۲ + ۴\cos^۲\alpha}{۱+\cos^۲\alpha} - \frac{(۱-\sin^۲\alpha)^۲ + ۴\sin^۲\alpha}{۱+\sin^۲\alpha} = \frac{۱+\cos^۴\alpha + ۲\cos^۲\alpha}{۱+\cos^۲\alpha} - \frac{۱+\sin^۴\alpha + ۲\sin^۲\alpha}{۱+\sin^۲\alpha} =$$

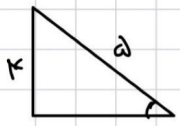
$$\frac{(۱+\cos^۲\alpha)^۲}{۱+\cos^۲\alpha} - \frac{(۱+\sin^۲\alpha)^۲}{۱+\sin^۲\alpha} = ۱+\cos^۲\alpha - (۱+\sin^۲\alpha) = \cos^۲\alpha - \sin^۲\alpha = (\cos\alpha+\sin\alpha)(\cos\alpha-\sin\alpha)$$

$\cot^۲\alpha$

-۷

$$\sin\left(\frac{۹\pi}{۲}+\alpha\right)\cos\left(\frac{۷\pi}{۲}-\alpha\right) - \tan\left(\alpha - \frac{۳\pi}{۲}\right) = -\cos\alpha(\sin\alpha) + \cot\alpha = -\frac{۳}{۵}\times\left(\frac{۴}{۵}\right) + \frac{۳}{۴} = \frac{-۹}{۲۰} + \frac{۳}{۴} = \frac{۲۷}{۱۰۰}$$

۱,۵



$\tan\alpha = \frac{۴}{۳}$ (سوم ناحیه) $\Rightarrow \sin\alpha = -\frac{۴}{۵}$

$\cot\alpha = \frac{۳}{۴}$ $\quad \cos\alpha = -\frac{۳}{۵}$

-۱

$$۳\cos۴\pi + \sqrt{۲}\sin\pi - \sqrt{۲}\sin\pi \xrightarrow{u=\frac{\pi}{۱۲}} ۳\cos\frac{۴\pi}{۱۲} + \sqrt{۲}\sin\frac{\pi}{۱۲} - \sqrt{۲}\cos\frac{\pi}{۱۲} = ۳\cos\frac{\pi}{۳} + \sqrt{۲}\sin\frac{\pi}{۴} - \sqrt{۲}\cos\frac{\pi}{۳} =$$

$\cos\frac{\pi}{۳} = \frac{۱}{۲}$

$$\frac{۳}{۲} + \sqrt{۲}\left(\sin\frac{\pi}{۱۲} - \cos\frac{\pi}{۴}\right) = \frac{۳}{۲} + ۲\sin\left(-\frac{\pi}{۶}\right) = \frac{۱}{۲}$$

$\sqrt{۲}\sin\left(\frac{\pi}{۱۲} - \frac{\pi}{۴}\right) = -\frac{\pi}{۶}$ $\qquad \sin\left(-\frac{\pi}{۶}\right) = -\frac{۱}{۲}$

۶

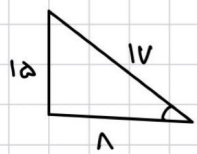
-۹

$$\tan\left(\frac{\alpha}{۲}\right) = \frac{\sin\alpha}{۱+\cos\alpha} = \frac{۱}{۴} \Rightarrow ۴\sin\alpha = ۱+\cos\alpha$$

$$\frac{۱-\cos\alpha}{\sin\alpha} = \frac{۱}{۴} \Rightarrow \sin\alpha = ۴ - ۴\cos\alpha \xrightarrow{\times ۴} ۴\sin\alpha = ۱۶ - ۱۶\cos\alpha \Rightarrow ۱۶ - ۱۶\cos\alpha = ۱+\cos\alpha$$

$$\Rightarrow ۱۷\cos\alpha = ۱۵ \Rightarrow \cos\alpha = \frac{۱۵}{۱۷}$$

۱



$\sin\alpha = \frac{۸}{۱۷}$

$\tan\alpha = \frac{۸}{۱۵}$

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{۸}{۱۵} - \frac{۸}{۱۷}}{\frac{۸}{۱۷} - \frac{۱۵}{۱۷}} = \frac{\frac{۱۹۱}{۱۲۷۵}}{\frac{۷}{۱۷}} = \frac{۱۹۱}{۵۷۵}$$

-۱۰ (ب)

$$\frac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow \frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} > 0 \Rightarrow \underbrace{\frac{\cos\alpha}{\sin^2\alpha}}_{\text{همواره مثبت}} > 0 \Rightarrow \cos\alpha > 0 \longrightarrow$$ α ناحیه اول یا چهارم

$$\sin 2\alpha = 2\sin\alpha\cos\alpha \Rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha$$

α ناحیه اول $\rightarrow$ $\cancel{2\sin\alpha} < \cancel{2\sin\alpha}\cos\alpha \Rightarrow \cos\alpha > 1$ غیر ممکن

ناحیه چهارم $\rightarrow$ $\cancel{2\sin\alpha} < \cancel{2\sin\alpha}\cos\alpha \overset{\sin\alpha<0}{\Longrightarrow} \cos\alpha < 1$ ✓

α در ناحیه چهارم قرار دارد.

۶)
$$\frac{\sin^4\alpha + 4(1-\sin^2\alpha)}{1+(1-\sin^2\alpha)} - \frac{\cos^4\alpha + 4(1-\cos^2\alpha)}{1+(1-\cos^2\alpha)}$$

$$= \frac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha} - \frac{(2-\cos^2\alpha)^2}{2-\cos^2\alpha} = 2-\sin^2\alpha - 2 + \cos^2\alpha$$

$$= \cos 2\alpha$$

۹)
$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{8}{15} \qquad \cos\alpha = \frac{15}{17}, \quad \sin\alpha = \frac{8}{17}$$

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{-16}{105}$$