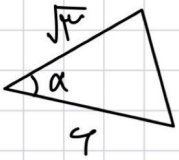
سازمان تکلیف شماره ۲۷ ـ یازدهم دختر A

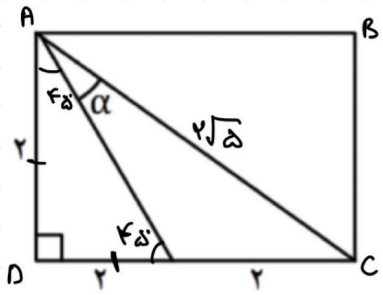
۱-



$S=\frac{1}{2}ab\sin\alpha \Rightarrow \frac{1}{2}\times\sqrt{3}\times 4\times\sin\alpha = 3\sqrt{3} \Rightarrow \sin\alpha = \frac{3}{2\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

$\alpha = 60^\circ \text{ یا } 120^\circ \Rightarrow \frac{120}{60} = 2$

۲-



$AC^2 = DC^2 + AD^2 \Rightarrow AC = 2\sqrt{5}$

$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\beta + \cot\alpha} \Rightarrow \frac{\cot\alpha - 1}{1+\cot\alpha} = \frac{1}{2} \Rightarrow$

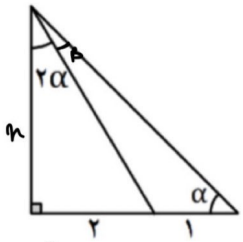
$\cot(\alpha+45^\circ) = \frac{2}{4} = \frac{1}{2}$

$2\cot\alpha - 2 = 1 + \cot\alpha \Rightarrow$

$\cot 45^\circ = 1$

$\cot\alpha = 3$

۳-



$\cot\alpha = \frac{3}{n}$

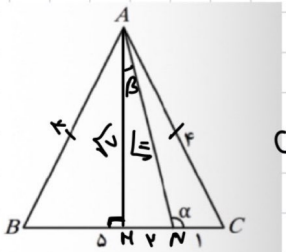
$\cot 2\alpha = \frac{n}{2} \Rightarrow \cot(\alpha+\alpha) = \frac{\cot\alpha\cot\alpha - 1}{\cot\alpha + \cot\alpha} \Rightarrow$

$\frac{\frac{9}{n^2} - 1}{\frac{6}{n}} = \frac{n}{2} \Rightarrow \frac{\frac{9-n^2}{n^2}}{\frac{6}{n}} = \frac{9-n^2}{6n} = \frac{n}{2} \Rightarrow 18 - 2n^2 = 6n^2$

$8n^2 = 18 \Rightarrow n^2 = \frac{18}{8} \Rightarrow n = \frac{3\sqrt{2}}{2\sqrt{2}} = \frac{3}{2}$

$\cot\alpha = \frac{3}{n} = \frac{3}{\frac{3}{2}} = 2$

۴-



$AB^2 = BH^2 + AH^2 \Rightarrow AH = \sqrt{7}$ ، $AN^2 = NH^2 + AH^2 \Rightarrow AN = \sqrt{11}$

α خارجی $= \beta + 90^\circ \Rightarrow \cos\alpha = -\sin\beta$ ، $\sin\alpha = \cos\beta$

$\Rightarrow \cos\alpha = -\frac{2}{\sqrt{11}}$ ، $\sin\alpha = \frac{\sqrt{7}}{4}$

$\Rightarrow \tan\alpha = \frac{\frac{\sqrt{7}}{4}}{-\frac{2}{\sqrt{11}}} = \frac{-\sqrt{7}}{2}$

-۵

$$۲\sin^۲\theta + \cos^۲\theta = \frac{۶}{۳}$$

$$\sin^۲\theta + \cos^۲\theta = ۱$$

$$\sin^۲\theta = \frac{۱}{۳} \Rightarrow \cos^۲\theta = \frac{۲}{۳} \Rightarrow \tan^۲\theta = \frac{\sin^۲\theta}{\cos\theta} = \frac{\frac{۱}{۳}}{\frac{۲}{۴}} = \frac{۱}{۲}$$

-۶

$$\frac{\sin^۴\alpha + ۴\cos^۲\alpha}{۱+\cos^۲\alpha} - \frac{\cos^۴\alpha + ۴\sin^۲\alpha}{۱+\sin^۲\alpha} =$$

$$\sin^۲\alpha = ۱ - \cos^۲\alpha$$

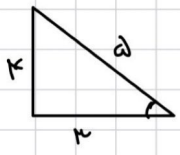
$$\cos^۲\alpha = ۱ - \sin^۲\alpha$$

$$\frac{(۱-\cos^۲\alpha)^۲ + ۴\cos^۲\alpha}{۱+\cos^۲\alpha} - \frac{(۱-\sin^۲\alpha)^۲ + ۴\sin^۲\alpha}{۱+\sin^۲\alpha} = \frac{۱+\cos^۴\alpha + ۲\cos^۲\alpha}{۱+\cos^۲\alpha} - \frac{۱+\sin^۴\alpha + ۲\sin^۲\alpha}{۱+\sin^۲\alpha} =$$

$$\frac{(۱+\cos^۲\alpha)^۲}{۱+\cos^۲\alpha} - \frac{(۱+\sin^۲\alpha)^۲}{۱+\sin^۲\alpha} = ۱+\cos^۲\alpha - (۱+\sin^۲\alpha) = \cos^۲\alpha - \sin^۲\alpha = (\cos\alpha+\sin\alpha)(\cos\alpha-\sin\alpha)$$

-۷

$$\sin\left(\frac{۹\pi}{۲}+\alpha\right)\cos\left(\frac{۷\pi}{۲}-\alpha\right) - \tan\left(\alpha - \frac{۳\pi}{۲}\right) = -\cos\alpha(\sin\alpha) + \cot\alpha = \frac{۳}{۵} \times \left(-\frac{۴}{۵}\right) \times \frac{۳}{۴} = -\frac{۹}{۲۵}$$



$\tan\alpha = \frac{۴}{۳}$ (ناحیه سوم) $\Rightarrow \sin\alpha = -\frac{۴}{۵}$

$\cot\alpha = \frac{۳}{۴}$ $\quad \cos\alpha = -\frac{۳}{۵}$

-۱

$$۳\cos۴\pi + \sqrt{۲}\sin\pi - \sqrt{۲}\sin\pi \xrightarrow{u=\frac{\pi}{۱۲}} ۳\cos\frac{۴\pi}{۱۲} + \sqrt{۲}\sin\frac{\pi}{۱۲} - \sqrt{۲}\cos\frac{\pi}{۱۲} = ۳\cos\frac{\pi}{۳} + \sqrt{۲}\sin\frac{\pi}{۴} - \sqrt{۲}\cos\frac{\pi}{۳} =$$

$$\frac{۳}{۲} + \sqrt{۲}\left(\sin\frac{\pi}{۱۲} - \cos\frac{\pi}{۴}\right) = \frac{۳}{۲} + ۲\sin\left(-\frac{\pi}{۶}\right) = \frac{۱}{۲}$$

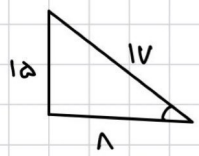
$\sqrt{۲}\sin\left(\frac{\pi}{۱۲} - \frac{\pi}{۴}\right) = -\frac{\pi}{۶}$ $\qquad \sin\left(-\frac{\pi}{۶}\right) = -\frac{۱}{۲}$

-۹

$$\tan\left(\frac{\alpha}{۲}\right) = \frac{\sin\alpha}{۱+\cos\alpha} = \frac{۱}{۴} \Rightarrow ۴\sin\alpha = ۱+\cos\alpha$$

$$\frac{۱-\cos\alpha}{\sin\alpha} = \frac{۱}{۴} \Rightarrow \sin\alpha = ۴ - ۴\cos\alpha \xrightarrow{\times ۴} ۴\sin\alpha = ۱۶ - ۱۶\cos\alpha \Rightarrow ۱۶ - ۱۶\cos\alpha = ۱+\cos\alpha$$

$$\Rightarrow ۱۷\cos\alpha = ۱۵ \Rightarrow \cos\alpha = \frac{۱۵}{۱۷}$$



$\sin\alpha = \frac{۸}{۱۷}$

$\tan\alpha = \frac{۱۵}{۸}$

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{۱۵}{۸} - \frac{۸}{۱۷}}{\frac{۸}{۱۷} - \frac{۱۵}{۱۷}} = \frac{\frac{۱۹۱}{۱۳۶}}{\frac{۷}{۱۷}} = \frac{۱۹۱}{۵۶}$$

$\frac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow \frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} > 0 \Rightarrow \frac{\cos\alpha}{\underbrace{\sin^2\alpha}_{\text{همواره مثبت}}} > 0 \Rightarrow \cos\alpha > 0 \xrightarrow{\text{ناحیه اول یا چهارم}}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha \Rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \xrightarrow{\text{ناحیه اول}} \cancel{2\sin\alpha} < \cancel{2\sin\alpha}\cos\alpha \Rightarrow \cos\alpha > 1$ غیر ممکن

$\xrightarrow{\text{ناحیه چهارم}} \cancel{2\sin\alpha} < \cancel{2\sin\alpha}\cos\alpha \overset{\sin\alpha<0}{\Rightarrow} \cos\alpha < 1$ ✓

α در ناحیه چهارم قرار دارد.