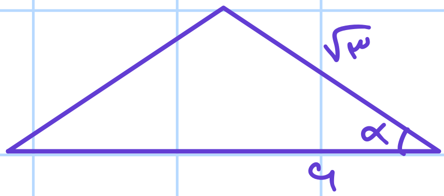
محمدطاها میرمقتدایی

۲۰

س۱

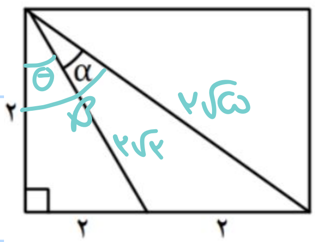


$\frac{\cancel{۹}^{۳}}{\cancel{۴}\,۵} = \frac{1}{\cancel{۲}} \times \sqrt{۳} \times \cancel{۶}^{۲} \times \sin\alpha \rightarrow \frac{۳}{۲\sqrt{۳}} = \sin\alpha$

$\frac{۳\sqrt{۳}}{۲\times۳} = \sin\alpha \rightarrow \sin\alpha = \frac{\sqrt{۳}}{۲} \rightarrow \alpha = ۶۰^\circ$ یا ۱۲۰°

$\frac{۱۲۰}{۶۰} = \boxed{۲}$

س۲



$\sin\beta = \frac{۴}{۲\sqrt{۵}}$ $\quad \cos\beta = \frac{۲}{۲\sqrt{۵}}$

$\sin\theta = \frac{۲}{۲\sqrt{۲}}$ $\quad \cos\theta = \frac{۲}{۲\sqrt{۲}}$

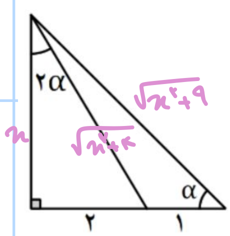
$\sin(\alpha) = \sin(\beta - \theta) = \sin\beta\cos\theta - \cos\beta\sin\theta$

$\sin\alpha = \frac{۴\sqrt{۵}}{\cancel{۲}\times۵} \times \frac{\cancel{۲}\sqrt{۲}}{۴} - \frac{\cancel{۲}\sqrt{۵}}{\cancel{۲}\times۵} \times \frac{۲\sqrt{۲}}{۴} = \frac{۴\sqrt{۱۰}}{۲۰} - \frac{۲\sqrt{۱۰}}{۲۰} = \frac{\sqrt{۱۰}}{۱۰}$

$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow \frac{۱۰}{۱۰۰} + \cos^2\alpha = 1 \rightarrow \cos^2\alpha = \frac{۹۰}{۱۰۰} \rightarrow \cos\alpha = \frac{۳\sqrt{۱۰}}{۱۰}$

$\cot\alpha = \frac{\frac{۳\sqrt{۱۰}}{\cancel{۱۰}}}{\frac{\sqrt{۱۰}}{\cancel{۱۰}}} = \boxed{۳}$

س۳



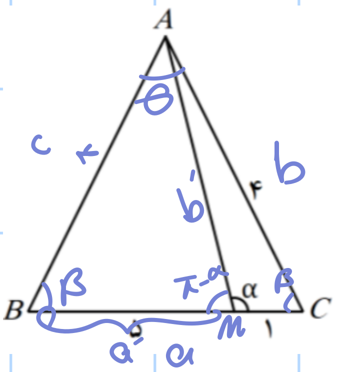
$\sin۲\alpha = \frac{۲}{\sqrt{x^2+۴}}$ $\quad \cos۲\alpha = \frac{x}{\sqrt{x^2+۴}} \rightarrow \cot۲\alpha = \frac{x}{۲}$

$\sin\alpha = \frac{x}{\sqrt{x^2+۹}}$ $\quad \cos\alpha = \frac{۳}{\sqrt{x^2+۹}} \rightarrow \tan\alpha = x/۳$

$$\frac{\cos 2\alpha}{\sin 2\alpha} \nearrow \cot 2\alpha = \frac{\frac{1-\tan^2\alpha}{\cancel{1+\tan^2\alpha}}}{\frac{2\tan\alpha}{\cancel{1+\tan^2\alpha}}} = \frac{1-\tan^2\alpha}{2\tan\alpha} \Rightarrow \frac{x}{2} = \frac{\frac{9-x^2}{9}}{\frac{2x}{3}}$$

$$\rightarrow \frac{x}{2} = \frac{9-x^2}{6x} \Rightarrow 3x^2 = 9 - x^2 \Rightarrow 4x^2 = 9 \Rightarrow x^2 = \frac{9}{4} \Rightarrow x = \frac{3}{2}$$

$$\cot\alpha = \frac{1}{\tan\alpha} = \frac{3}{x} \Rightarrow \boxed{\cot\alpha = 2}$$



$\sin\alpha = \sin(\pi-\alpha)$ $\cos\alpha =$

24

$$c^2 = a^2 + b^2 - 2ab\cos\beta \Rightarrow \cancel{16} = \cancel{16} + 36 - 2\times 6\times 4\cos\beta \Rightarrow$$

$$\cancel{18}^{3} = \cancel{6}\times 4\cos\beta \Rightarrow \cos\beta = \frac{3}{4}$$

$$b'^2 = c^2 + a'^2 - 2ac\cos\beta \rightarrow b'^2 = 16 + 25 - 2\times 4\times 5\times\frac{3}{4} \Rightarrow b'^2 = 11$$

$$16 = \cancel{11+1}^{12} - 2\sqrt{11}\cos\alpha \rightarrow \cancel{4}^{2} = -\cancel{2}\sqrt{11}\cos\alpha \rightarrow \cos\alpha = \frac{-2}{\sqrt{11}}$$

$$\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow \frac{4}{11} + \sin^2\alpha = 1 \rightarrow \sin\alpha = \pm\frac{\sqrt{7}}{\sqrt{11}}$$

غ ق ق چون α < ۱۸۰ است

$$\tan\alpha = \frac{\frac{\sqrt{7}}{\cancel{\sqrt{11}}}}{\frac{-2}{\cancel{\sqrt{11}}}}, \quad \boxed{\frac{-\sqrt{7}}{2}}$$

$$2\sin^2 x + \cos^2 x = \frac{5}{4} \Rightarrow \sin^2 x = \frac{1}{4}$$

$$\sin^2 x + \sin^2 x$$

$$\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = \frac{3}{4}$$

$$\tan^2 x = \frac{\frac{1}{4}}{\frac{3}{4}} = \boxed{\frac{1}{3}}$$

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} \Rightarrow$$

$$\sin^4\alpha = (1-\cos^2\alpha)^2 \qquad \cos^4\alpha = (1-\sin^2\alpha)^2$$

$$1+\cos^4\alpha - 2\cos^2\alpha + 4\cos^2\alpha \Rightarrow \frac{\cos^4\alpha + 2\cos^2\alpha + 1}{1+\cos^2\alpha} \Rightarrow \frac{(1+\cos^2\alpha)^2}{\cancel{1+\cos^2\alpha}}$$

$$1+\sin^4\alpha - 2\sin^2\alpha + 4\sin^2\alpha \Rightarrow \frac{\sin^4\alpha + 2\sin^2\alpha + 1}{1+\sin^2\alpha} \Rightarrow \frac{(1+\sin^2\alpha)^2}{\cancel{1+\sin^2\alpha}}$$

$$1+\cos^2\alpha - 1 - \sin^2\alpha = \cos^2\alpha - \sin^2\alpha = \boxed{\cos 2\alpha}$$

$$\underbrace{\sin\left(\frac{9\pi}{2}+\alpha\right)}_{\cos(\alpha)}\underbrace{\cos\left(\frac{7\pi}{2}-\alpha\right)}_{-\sin(\alpha)}-\underbrace{\tan\left(\alpha-\frac{3\pi}{2}\right)}_{-\cot(\alpha)}=$$ ۴۷

$$\cos\alpha \times -\sin\alpha + \cot\alpha$$

$$\tan\alpha = \frac{4}{3} \qquad \cot\alpha = \frac{3}{4}$$

$$1+\tan^2 = \frac{1}{\cos^2} \Rightarrow \frac{25}{9} = \frac{1}{\cos^2\alpha} \Rightarrow \cos\alpha = \frac{3}{5} \qquad \sin\alpha = \frac{4}{5}$$

$$\frac{3}{5}\times\frac{-4}{5}+\frac{3}{4} = \frac{-12}{25}+\frac{3}{4} = \frac{75-48}{100} = \boxed{\frac{27}{100}}$$

$$\frac{1}{\cancel{12}}\times \cancel{180}^{15} = 15^\circ$$ ۴۸

$$3\cos 60 + \sqrt{2}\sin 15 - \sqrt{2}\cos 15 \rightarrow \frac{3}{2} - \sqrt{2}\left(\frac{\cos^2 15 - \sin^2 15}{\cos 15 + \sin 15}\right)$$

$$\Rightarrow \frac{3}{2} - \cancel{\sqrt{2}}\left(\frac{\cos \pi/6}{\cancel{\sqrt{2}}\,\sin\left(\frac{\pi}{12}+\frac{\pi}{4}\right)}\right) = \frac{3}{2} - \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \frac{3}{2} - 1 = \boxed{\frac{1}{2}}$$

$$\sin\left(\frac{\pi}{12}+\frac{\pi}{4}\right) \rightarrow \sin \pi/3$$

$$\tan\frac{\alpha}{2} = \frac{1}{4} \Rightarrow \tan\alpha = \frac{2\tan\alpha/2}{1-\tan^2\alpha/2}$$ ۴۹

$$\tan\alpha = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} \Rightarrow \tan\alpha = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15}$$

$$1+\tan^2\alpha=\frac{1}{\cos^2\alpha} \Rightarrow 1+\frac{64}{225}=\frac{1}{\cos^2\alpha} \Rightarrow \frac{289}{225}=\frac{1}{\cos^2\alpha} \Rightarrow \cos\alpha=\frac{15}{17}$$

$$\tan\alpha=\frac{\sin\alpha}{\cos\alpha} \Rightarrow \frac{8}{15}=\frac{\sin\alpha}{\frac{15}{17}} \Rightarrow \sin\alpha=8/17$$

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=\frac{\frac{8(17-15)}{15\times 17}}{\frac{8-15}{17}}=\frac{\frac{8\times 2}{15}}{\frac{-7}{1}}\boxed{=-\frac{16}{105}}$$

س ۱۵

$0<\frac{\cot\alpha}{\sin\alpha} \Rightarrow \cot\alpha, \sin\alpha>0 \Rightarrow \cos\alpha>0$ ربع اول

$\cot\alpha<0\ ,\ \sin\alpha<0 \Rightarrow \cos\alpha>0$ ربع چهارم

۲

$2\sin\alpha<\sin 2\alpha \Rightarrow \cancel{2}\sin\alpha<\cancel{2}\sin\alpha\cos\alpha \Rightarrow \sin\alpha<\sin\alpha\cos\alpha \Rightarrow$

$-1\leq\cos\alpha\leq 1$

$\sin\alpha-\sin\alpha\cos\alpha<0 \Rightarrow \sin\alpha(1-\cos\alpha)<0 \Rightarrow \sin\alpha<0$

همیشه مثبت

در ربع چهارم قرار دارد