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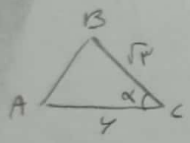
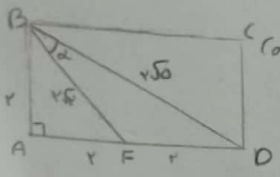


$\frac{1}{2}ab\sin\alpha = S \Rightarrow \frac{7}{2} = \frac{1}{2}\sqrt{2}\times 7\times\sin\alpha$

$\frac{\sqrt{2}}{2} = \sin\alpha \Rightarrow \alpha = 45^\circ$ یا $\alpha = 135^\circ$ $\Rightarrow \frac{135}{45} = 3$ برابر



$\cot\alpha$?

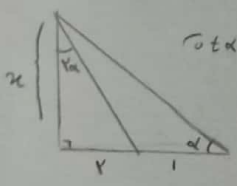
$BD^2 = 2^2 + 4^2 \Rightarrow BD^2 = 20 \Rightarrow BD = \sqrt{20} = 2\sqrt{5}$

$2 = \sqrt{\frac{(2\sqrt{2})^2}{8} + \frac{(2\sqrt{5})^2}{20} - \frac{2(2\sqrt{2})(2\sqrt{5})\cos\alpha}{4\sqrt{10}}} \Rightarrow 2 = \sqrt{28 - 8\sqrt{10}\cos\alpha} \Rightarrow 4 = 28 - 8\sqrt{10}\cos\alpha$

$24 = 8\sqrt{10}\cos\alpha$

$\cos\alpha = \frac{3}{\sqrt{10}}$

$\tan^2\alpha + 1 = \frac{1}{\cos^2\alpha} \Rightarrow \tan^2\alpha = \frac{1}{\frac{9}{10}} - 1 \Rightarrow \tan^2\alpha = \frac{10}{9} - \frac{9}{9} = \frac{1}{9} \Rightarrow \cot\alpha = 3$

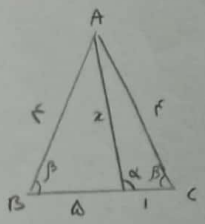


$\cot\alpha = ?$

$x = 2\sqrt{1+2^2} = 2\sqrt{5}$

$4 = \sqrt{1+4-2\times 2\sqrt{4}\times 2\sqrt{5}\cos\alpha} \Rightarrow 4 = 28 - 8\sqrt{10}\cos\alpha \Rightarrow \cos\alpha = \frac{3}{\sqrt{10}} \Rightarrow$

$\cot\alpha = 3$



$x = \sqrt{4^2+5^2-2(4)(5)\cos\beta}$

$x = \sqrt{4^2+1^2-2(4)(1)\cos\beta}$

$\Rightarrow 41 - 40\cos\beta = 17 - 8\cos\beta$

$24 = 24\cos\beta \Rightarrow \frac{3}{4} = \cos\beta \Rightarrow x = 17 - 8\cos\beta = 11 \Rightarrow x = 11$

$4 = \sqrt{11^2+1^2-2(11)(1)\cos\alpha} \Rightarrow 14 = 122 - 22\cos\alpha \Rightarrow 10 = 22\cos\alpha \Rightarrow \cos\alpha = \frac{2}{\sqrt{11}}$

$\tan^2\alpha + 1 = \frac{1}{\frac{4}{11}} \Rightarrow \frac{11}{4} - 1 = \tan^2\alpha \Rightarrow \tan^2\alpha = \frac{7}{4} \Rightarrow \tan\alpha = -\frac{\sqrt{7}}{2}$

$2\sin^2 x + \cos^2 x = \frac{4}{3} \Rightarrow \sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} = \frac{4}{3} \Rightarrow \sin^2 x = \frac{1}{3} \Rightarrow \frac{7}{15} + \cos^2 x = \frac{4}{3} \Rightarrow$

$\cos^2 x = \frac{4}{3} - \frac{7}{15} = \frac{13}{15}$

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{13}{15}} = \frac{1}{13}$

$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{\sin^2\alpha+1} \Rightarrow \frac{(1-\cos^2\alpha)^2 + 4\cos^2\alpha}{\cos^2\alpha+1} - \frac{(1-\sin^2\alpha)^2 + 4\sin^2\alpha}{\sin^2\alpha+1} = \frac{1-2\sin^2\alpha+\sin^4\alpha+4\sin^2\alpha}{\sin^2\alpha+1} =$

$\frac{1-2\cos^2\alpha+\cos^4\alpha+4\cos^2\alpha}{\cos^2\alpha+1} = \frac{(\cos^2\alpha+1)^2}{\cos^2\alpha+1} = \cos^2\alpha+1$

$\cos^2\alpha + 1 - \sin^2\alpha - 1 = \cos 2\alpha$

$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{\pi}{2}-\alpha\right)-\tan\left(\alpha-\frac{3\pi}{2}\right)=(+\cos\alpha)\times(-\sin\alpha)+\cot\alpha \Rightarrow -\cos\alpha\sin^2\alpha-\cos\alpha \Rightarrow -\cos\alpha(\sin^2\alpha+1)$

$-\cos\alpha\sin\alpha$; $\cot\alpha=\frac{\cos\alpha}{\sin\alpha}$; $\frac{\cos\alpha}{\sin\alpha}=\tan\alpha$ (1)

(1)

$-\tan\alpha(\sin^2\alpha+1)$:

$-\frac{4}{3}\left(\frac{16}{25}+1\right)=-\frac{4}{3}\times\frac{41}{25}=-\frac{164}{75}$

$\cot\alpha=\frac{3}{4} \Rightarrow \frac{9}{16}+1=\frac{1}{\sin^2\alpha} \Rightarrow \sin^2\alpha=\frac{16}{25}$

$\frac{-3}{5}\times\frac{4}{5}+\frac{3}{4}=\frac{27}{100}$

$\frac{2\cos\frac{\pi}{3}}{\frac{1}{2}}+\sqrt{2}\sin\frac{\pi}{16}-\sqrt{2}\cos\frac{\pi}{16} \xrightarrow{\times\sqrt{2}} \frac{2\sqrt{2}}{2}+\frac{\sin\frac{\pi}{8}}{\frac{1}{2}}-\frac{\cos\frac{\pi}{8}}{\frac{\sqrt{2}}{2}}=\frac{2}{2}-1=\frac{1}{2}$ (1)

(5)

$\tan\frac{\alpha}{2}=\frac{1}{4} \Rightarrow \cos\frac{\alpha}{2}=\frac{4}{\sqrt{17}}$; $\sin\frac{\alpha}{2}=\frac{1}{\sqrt{17}}$ $\Rightarrow \sin\alpha=2\sin\frac{\alpha}{2}\times\cos\frac{\alpha}{2}=2\times\frac{4}{\sqrt{17}}\times\frac{1}{\sqrt{17}}=\frac{8}{17}$ (9)

$\sin\alpha=\frac{8}{17}$, $\cos=\frac{15}{17}$

$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{8}{15}-\frac{1}{17}}{\frac{8-15}{17}}=-\frac{17}{105}$

(5)

$2\sin\alpha<\sin2\alpha$ (10)

$2\sin\alpha<2\sin\alpha\cdot\cos\alpha \Rightarrow 2\sin\alpha-2\sin\alpha\cdot\cos\alpha<0 \Rightarrow 2\sin\alpha\times(1-\cos\alpha)<0 \Rightarrow \sin\alpha<0$

$2\sin\alpha\cdot\cos\alpha$; $(1-\cos\alpha)>0$; $\cos\alpha>-1$

$\frac{\cot\alpha}{\sin\alpha}>0 \Rightarrow \frac{\cos\alpha}{\sin^2\alpha}>0 \Rightarrow \cos\alpha>0$

$\pi<\alpha<2\pi$ (9)