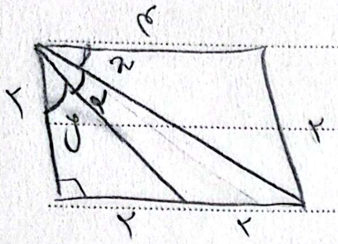
Subject : (   )   Date : ....

$\tan y = \frac{2}{2} = 1$

$\tan x = \frac{2}{4} = 1/2$

۲۰

حسنا رجبی ، شفیع ۲۷ یازدهم دختر A.



۱۳)

$\alpha + x + y = 90 \to \alpha = 90 - (x+y)$

$\cot\alpha = \cot(\pi/2 - (x+y)) = \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \times \tan y} = \frac{1/2 + 1}{1 - 1/2} = \frac{3/2}{1/2} = \boxed{3}$

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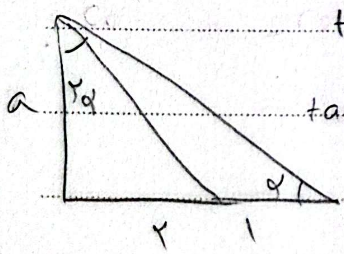
۱۲)

$S = 1/2\, a \times b \times \sin\alpha \to 1/2 \times \sqrt{3} \times 6 \times \sin\alpha = 3\sqrt{3} \times \sin\alpha = 4{,}5$

$\to \sin\alpha = \frac{1{,}5}{\sqrt{3}} = \frac{3/2}{\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \to \alpha = \frac{\pi}{3}, \quad \alpha = \frac{2\pi}{3}$ } ۲ بار

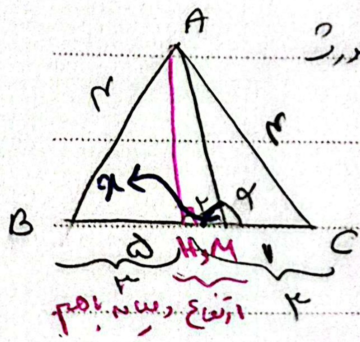


۱۳)

$\tan\alpha = \frac{a}{3} \to \cot\alpha = \frac{3}{a} = \frac{3}{3/2} = \frac{6}{3} = \boxed{2}$

$\tan 2\alpha = \frac{2}{a} = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{\frac{2a}{3}}{1 - a^2/9} = \frac{\cancel{6}a/3}{\frac{9-a^2}{9}} = \frac{\cancel{2}}{a}$

$3a^2 = \frac{9-a^2}{\cancel{4}} \to a^2 = 9/4 \to a = 3/2$



۱۴)

فیثاغورث : $(AH)^2 + (\overset{9}{CH})^2 = (\overset{16}{AC})^2 \to AH = \sqrt{7}$

$\cot\alpha = -\cot(\pi - \alpha) = -\frac{2}{\sqrt{7}} \Rightarrow \tan\alpha = \frac{-\sqrt{7}}{2}$

۱۵) $۲\sin^۲ + \cos^۲ = \sin^۲ + ۱ = ۴/۳ \rightarrow \sin^۲ = ۱/۳ \Rightarrow \cos^۲ = ۲/۳$

$\tan^۲ = \dfrac{۱/۳}{۲/۳} = \boxed{۱/۲}$

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۱۶) $\dfrac{\sin^۴ + ۴\cos^۲}{۱ + \cos^۲}$ ① $- \dfrac{\cos^۴ + ۴\sin^۲}{۱ + \sin^۲}$ ②

$\dfrac{\sin^۴ + ۴(۱ - \sin^۲)}{۱ + \underbrace{\cos^۲}_{۱ - \sin^۲}} = \dfrac{\sin^۴ + ۴ - ۴\sin^۲}{۲ - \sin^۲} = \dfrac{(۲ - \sin^۲)^۲}{۲ - \sin^۲} = ۲ - \sin^۲$

②: $\dfrac{\cos^۴ + ۴(۱ - \cos^۲)}{۱ + \underbrace{\sin^۲}_{۱ - \cos^۲}} = \dfrac{\cos^۴ + ۴ - ۴\cos^۲}{۲ - \cos^۲} = \dfrac{(۲ - \cos^۲)^۲}{۲ - \cos^۲} = ۲ - \cos^۲$

$۲ - \sin^۲ - ۲ + \cos^۲ = \cos^۲ - \sin^۲ = \boxed{\cos ۲\alpha}$

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۱۷) $\sin(\overset{\pi/۲}{\frac{۹\pi}{۲}} + \alpha) \times \cos(\overset{۳\pi/۲}{\frac{۷\pi}{۲}} - \alpha) - \tan(\underbrace{\alpha - \frac{۳\pi}{۲}}_{})$

$\cos\alpha \times -\sin\alpha + \cot\alpha =$ $\qquad -(۳\pi/۲ - \alpha) \rightarrow \tan(-\theta) = -\tan\theta$

$\Rightarrow -\tan(\alpha - ۳\pi/۲) = +\tan(۳\pi/۲ - \alpha)$

$\rightarrow -\dfrac{۳}{۵} \times +\dfrac{۴}{۵} + \dfrac{۳}{۴} = \dfrac{-۱۲}{۲۵} + \dfrac{۳}{۴} = \dfrac{-۴۸ + ۷۵}{۱۰۰} = \boxed{\dfrac{۲۷}{۱۰۰}}$

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۱۸) * $\sin\alpha - \cos\alpha = \sqrt{۲}\sin(\alpha - \frac{\pi}{۴}) \Rightarrow \sqrt{۲}(\sin x - \cos x) = ۲ \times \sin(\frac{\pi}{۱۲} - \frac{\pi}{۴})$

$= ۲\sin(-\frac{\pi}{۶}) = ۲ \times \frac{-۱}{۲} = (-۱)$ $\longrightarrow \frac{۳}{۲} - ۱ = (\frac{۱}{۲})$

$۳\cos ۴(\frac{\pi}{۱۲}) = ۳\cos(\frac{\pi}{۳}) = ۳ \times \frac{۱}{۲} = (\frac{۳}{۲})$

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۹) تانژانت تعریف شده است پس: $\frac{\alpha}{2} \neq k\pi + \frac{\pi}{2}$

$$\sin\alpha = \frac{2\tan(\frac{\alpha}{2})}{1+\tan^2(\frac{\alpha}{2})} = \frac{2\times\frac{1}{4}}{1+\frac{1}{16}} = \frac{1}{\cancel{2}} \times \frac{\cancel{16}^{8}}{17} = \boxed{\frac{8}{17}}$$

$$\cos\alpha = \frac{1-\tan^2(\frac{\alpha}{2})}{1+\tan^2(\alpha/2)} = \frac{1-\frac{1}{16}}{1+\frac{1}{16}} = \frac{15}{\cancel{16}} \times \frac{\cancel{16}}{17} = \boxed{\frac{15}{17}}$$

$$\tan\alpha = \frac{8}{17} \times \frac{17}{15} = \boxed{\frac{8}{15}}$$

$$\rightarrow \frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{16}{255}}{-\frac{7}{17}} = \frac{16}{\cancel{255}_{15}} \times \frac{-1}{\cancel{17}} \cdot \frac{\cancel{-17}}{7} = \boxed{\frac{-16}{105}}$$

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۱۰) $2\sin\alpha < \sin 2\alpha \rightarrow \cancel{2}\cancel{\sin\alpha} < \cancel{2}\cancel{\sin\alpha}\cos\alpha \xrightarrow{\sin>0} \cos\alpha > 1$ ✗

ناحیه سوم و چهارم $\boxed{\sin<0} \rightarrow \cos\alpha < 1$ ✓

$\frac{\cot\alpha}{\sin\alpha} > 0 \rightarrow \frac{\cos\alpha}{\sin^2\alpha} > 0 \rightarrow \boxed{\cos\alpha > 0} \Longrightarrow$ ناحیه چهارم