

$$\frac{\sin^2 \alpha + E \cos^2 \alpha}{1 + \cos^2 \alpha} = \frac{\cos^2 \alpha + E \sin^2 \alpha}{\sin^2 \alpha + 1} \Rightarrow \frac{(1 - \cos^2 \alpha)^E + E \cos^2 \alpha}{\cos^2 \alpha + 1} = \frac{(1 - \sin^2 \alpha)^E + E \sin^2 \alpha}{\sin^2 \alpha + 1}$$

$$\frac{1 + \cos^2 \alpha + E \cos^2 \alpha + E \cos^2 \alpha}{\cos^2 \alpha + 1} = \frac{(\cos^2 \alpha + 1)^E}{\cos^2 \alpha + 1} = \cos^2 \alpha + 1 \quad \Big/ \quad \frac{1 - \sin^2 \alpha + \sin^2 \alpha + E \sin^2 \alpha + E \sin^2 \alpha}{\sin^2 \alpha + 1} = \frac{(\sin^2 \alpha + 1)^E}{\sin^2 \alpha + 1} = \sin^2 \alpha + 1$$

$$\cos^2 \alpha + \sin^2 \alpha = \cos^2 \alpha - \sin^2 \alpha = \cos 2\alpha$$

(8)

$\tan \alpha = \frac{E}{V} \rightarrow$

$$\sin \alpha = \frac{E}{\sqrt{E^2 + V^2}}, \quad \cos \alpha = \frac{V}{\sqrt{E^2 + V^2}}$$

$$\sin\left(\frac{\pi}{4} + \alpha\right) \times \cos\left(\frac{\pi}{4} - \alpha\right) - \tan\left(\alpha - \frac{\pi}{4}\right) = \cos \alpha \times \sin \alpha + \cot \alpha = -x \frac{V}{E} \times \frac{E}{V} + \frac{V}{E}$$

$$\Rightarrow \frac{-V}{V} + \frac{V}{E} = \frac{-E + V}{E} = \frac{-1 + V}{E}$$

(9)

$$x = \frac{\pi}{12} = \sqrt{\cos\left(\frac{E \times \pi}{V \times \pi}\right)} + \sqrt{\sin\left(\frac{\pi}{12} - \cos\left(\frac{\pi}{12}\right)\right)} = \frac{V}{V} + \sqrt{x \times (-x) \times \frac{1}{x}} = \frac{V}{V} - 1 = \frac{1}{V}$$

$$\frac{1 - \frac{V}{V}}{12} = \frac{-V}{12} = \frac{-\pi}{12}$$

(10)

$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{E} \rightarrow$

$$\left. \begin{aligned} \cos\left(\frac{\alpha}{2}\right) &= \frac{E}{\sqrt{1 + E^2}} \\ \sin\left(\frac{\alpha}{2}\right) &= \frac{1}{\sqrt{1 + E^2}} \end{aligned} \right\} \Rightarrow \sin \alpha = \frac{1}{\sqrt{1 + E^2}} \Rightarrow \sin \alpha = \frac{1}{V}$$

$$\Rightarrow \cos \alpha = \frac{E}{V} \Rightarrow \frac{1}{10} - \frac{1}{10} = \frac{10 - 10}{10} = \frac{-10}{10} = -1$$

(11)

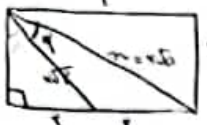
$$r \sin \alpha < \frac{\sin^2 \alpha}{\sin \alpha \cdot \cos \alpha} \Rightarrow r \sin \alpha - r \sin \alpha \cdot \cos \alpha < \cdot \rightarrow r \sin \alpha (1 - \cos \alpha) < \cdot \Rightarrow r \sin \alpha < \cdot$$

$$\Rightarrow \frac{\cot \alpha}{\sin \alpha} > \cdot \Rightarrow \frac{\cos \alpha}{\sin^2 \alpha} > \cdot \Rightarrow \frac{\cos \alpha}{\sin^2 \alpha} > \cdot \Rightarrow \frac{E}{V} > \cdot$$

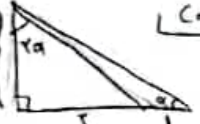
(12)

$s = \frac{1}{2} bc \sin \alpha \Rightarrow \frac{1}{2} \times \sqrt{3} \times \sqrt{3} \times \sin \alpha = \frac{\sqrt{3}}{2} \Rightarrow \sin \alpha = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow \sin \alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 120^\circ, 60^\circ$
 $\Rightarrow \frac{120^\circ}{2} = 60^\circ$

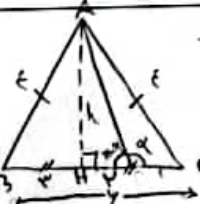
①

$x = \sqrt{1 + \sqrt{3}} = \sqrt{2}$ (طبق قضیه پیتاگوراس)

 $r = \sqrt{1 + \sqrt{3} - \sqrt{3} \sqrt{1 + \sqrt{3}} \cos \alpha} \Rightarrow r = \sqrt{1 + \sqrt{3} - \sqrt{3} \cos \alpha} \Rightarrow \frac{\sqrt{3}}{\sqrt{1 + \sqrt{3}}} = \cos \alpha \Rightarrow \cos \alpha = \frac{\sqrt{3}}{\sqrt{1 + \sqrt{3}}}$
 $\Rightarrow \cot \alpha = \frac{r}{1} = \frac{\sqrt{3}}{1} = \sqrt{3}$

②

$\cot \alpha = ? \Rightarrow \cot \alpha = \frac{r}{n} \quad , \quad \cot 2\alpha = \frac{n}{x} \Rightarrow \cot 2\alpha = \frac{\cot \alpha - \tan \alpha}{1 + \cot \alpha \tan \alpha}$

 $\Rightarrow \cot \alpha = n + \tan \alpha \Rightarrow \frac{r}{1} = n + \frac{1}{n} \Rightarrow r = n + \frac{1}{n} \Rightarrow n^2 = \frac{r}{2} \Rightarrow n = \frac{r}{2}$
 $\Rightarrow \cot \alpha = \frac{r}{\frac{r}{2}} = 2$

③

$\tan \alpha = ?$
 ارتفاع مثلث مستوی الساقین معلوم می‌باشد و طبق قضیه پیتاگوراس داریم:
 $h = \sqrt{r^2 - a^2} = \sqrt{r^2 - \frac{r^2}{4}} = \frac{\sqrt{3}r}{2} \Rightarrow (r^2 = 1 + h^2)$

 $\Rightarrow \tan(\pi - \alpha) = -\tan \alpha = \frac{h}{\frac{1}{2}} = \frac{\sqrt{3}r}{2} \Rightarrow \tan \alpha = \frac{-\sqrt{3}r}{2}$

④

$\frac{r \sin^2 n + \cos^2 n = \frac{r}{2}}{\sin^2 n + \cos^2 n = 1} \Rightarrow \sin^2 n + 1 = \frac{r}{2} \Rightarrow \sin^2 n = \frac{r}{2} - 1 \Rightarrow \cos^2 n = 1 - \sin^2 n = 1 - \left(\frac{r}{2} - 1\right) = \frac{2-r}{2}$
 $\Rightarrow \tan^2 n = \frac{\sin^2 n}{\cos^2 n} = \frac{\frac{r}{2} - 1}{\frac{2-r}{2}} = \frac{r-2}{2-r} = -1$

⑤