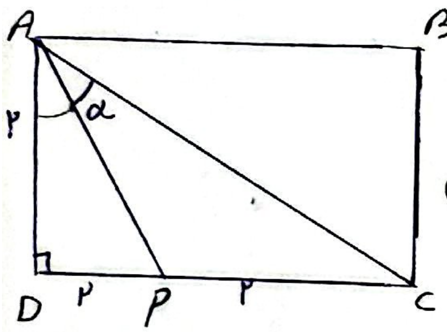
[illegible] یازدهم دختر A — [illegible]

$\sqrt{3}\,،\,6\,،\,\hat{\alpha}$ $\quad S = 4.5 \quad$ بیشترین مقدار $= 120^\circ \quad$ کمترین مقدار $= 60^\circ$

$$S_{\text{مثلث}} = \frac{\sqrt{3} \times \overset{3}{\cancel{6}} \times \sin\hat{\alpha}}{\cancel{2}} = 4.5 \to 3\sqrt{3} \times \sin\alpha = 4.5 \to \sin\alpha = \frac{\overset{3/2}{\cancel{4.5}}}{\cancel{3}\sqrt{3}} = \frac{\sqrt{3}}{2} \quad (1)$$

$$\Rightarrow \hat{\alpha} = 60^\circ \text{ یا } \hat{\alpha} = 120^\circ \qquad \frac{120}{60} = 2 \;\; \boxed{\text{برابر}} \leftarrow \text{جواب}$$

$\cot\alpha$:



$AD^2 + DP^2 = AP^2 \Rightarrow \sqrt{4+4} = 2\sqrt{2} = AP$

$AC = \sqrt{4+16} = 2\sqrt{5}$

(1) $S_{APC} = 2 \times 2 \times \frac{1}{2} = 2$

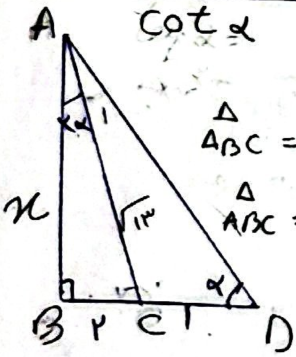
(2) $S_{APC} = \frac{2\sqrt{2} \times 2\sqrt{5} \times \sin\alpha}{2} = 2 \to 4\sqrt{10} \times \sin\alpha = 4 \quad (2)$

$\sin\alpha = \frac{1}{\sqrt{10}}$

$1 + \cot^2 = \frac{1}{\sin^2} \to \cot^2 = 10 - 1 = 9 \sim \cot\alpha = \boxed{+3} \checkmark$

-3 ✗ غلط چون از 90° درجه کمتره

$\cot\alpha$

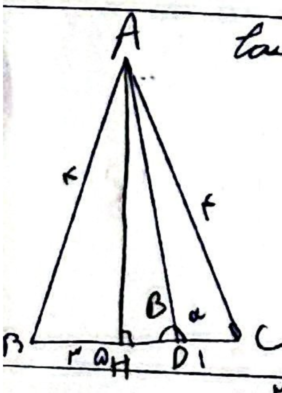


$$\left.\begin{aligned} \overset{\Delta}{ABC} &= \tan 2\alpha = \frac{2}{x} \\ \overset{\Delta}{ABC} &= \tan\alpha = \frac{x}{3} \end{aligned}\right\} \quad \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2 \times \frac{x}{3}}{1 - \frac{x^2}{9}} = \frac{2}{x}$$

$$\Rightarrow \frac{\frac{2x}{3}}{\frac{9-x^2}{9}} = \frac{2}{x} \to \frac{6x^2}{9} = \frac{18-x^2}{18} \sim 6x^2 = 108 - 6x^2 \quad (3)$$

$$\to 12x^2 = 108 \sim x^2 = 9 \to x = 3$$

$\overset{\Delta}{ABC} = \sqrt{3^2 + 2^2} = AC \to \sqrt{13} \qquad \cot\alpha = \frac{1}{\sqrt{13}}$ جواب



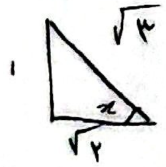
$\tan\alpha$:

$\hat{\alpha} + \hat{B} = 180 \to \tan\alpha = -\tan B \to \tan\alpha = -\frac{\sqrt{7}}{2}$

$\left.\begin{aligned} BH &= HC = 3 \\ HD &= 3 - 1 = 2 \end{aligned}\right\} \to AD = \sqrt{7+4} = \sqrt{11} \sim \tan B = \frac{\sqrt{7}}{2} \quad (4)$

$AH^2 + BH^2 = AB^2 \to AH^2 = 16 - 9 = 7 \to AH = \sqrt{7}$

$\tan x = ?$ $\quad 2\sin^2 x + \cos^2 x = \frac{4}{3} \quad = \sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} = \frac{4}{3} \to \sin^2 x = \frac{1}{3} \to \sin x = \pm\frac{1}{\sqrt{3}}$



$\sim \tan x = \pm\frac{1}{\sqrt{2}} \to \tan^2 x = \frac{1}{2}$ جواب $\quad (5)$

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}$$ $$\frac{\sin^4\alpha+4(1-\sin^2\alpha)}{1+1-\sin^2\alpha} = \frac{\sin^4\alpha+4-4\sin^2\alpha}{2-\sin^2\alpha} = \frac{(\sin^2\alpha-2)^2}{-(\sin^2\alpha-2)}$$ (1)

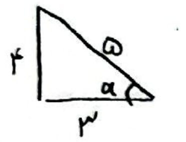
کسر اول $=(2-\sin^2\alpha)$ / کسر دوم: $\frac{\cos^4\alpha+4(1-\cos^2\alpha)}{1+1-\cos^2\alpha} = \frac{\cos^4\alpha+4-4\cos^2\alpha}{2-\cos^2\alpha} = \frac{(\cos^2\alpha-2)^2}{-(\cos^2\alpha-2)}$ (4)

کسر دوم $=(2-\cos^2\alpha)$ $\Big\} \Rightarrow (2-\sin^2\alpha)-(2-\cos^2\alpha) = -\sin^2\alpha+\cos^2\alpha \to \frac{-1+\cos2\alpha+1+\cos2\alpha}{2}$

$\Rightarrow \frac{2\cos2\alpha}{2} = \boxed{\cos2\alpha}$ ← جواب

$\tan\alpha = \frac{4}{3}$ $\quad \alpha \leftarrow$ در ربع سوم

$$\sin(\underbrace{\tfrac{9\pi}{2}}_{4\pi+\frac{\pi}{2}}+\alpha)\cos(\underbrace{\tfrac{7\pi}{2}}_{3\pi+\frac{\pi}{2}}-\alpha) - \tan(\alpha-\tfrac{3\pi}{2}) = (\cos\alpha)\times(-\sin\alpha) - (-\cot\alpha) =$$



$$\Rightarrow -\cos\alpha\sin\alpha + \cot\alpha = -\frac{3}{5}\times\frac{4}{5} + \frac{3}{4} = -\frac{12}{5}+\frac{3}{4} = \frac{-48+15}{20}$$

$$= \boxed{\frac{-33}{20}}$$ ← جواب

$x = \frac{\pi}{12}$ $\quad (3\cos4x + \sqrt{2}\sin x - \sqrt{2}\cos x) = 3\cos 4\times\frac{\pi}{12} + \sqrt{2}(\sin x - \cos x)$

$= 3\cos\frac{\pi}{3} + \sqrt{2}(\sin x - \cos x) \xrightarrow{(1)} 3\times\frac{1}{2} + \sqrt{2}(\sqrt{2}\sin(x-\frac{\pi}{4})) = \frac{3}{2} + 2(-\frac{1}{2}) = \frac{1}{2}$ ← جواب

(1) $\sin x - \cos x = \sqrt{2}\sin(x-\frac{\pi}{4})$

$\Rightarrow$

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$\tan(\frac{\alpha}{2}) = \frac{1}{4}$ $\quad \frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \dfrac{\frac{2\tan(\frac{\alpha}{2})}{1-\tan^2(\frac{\alpha}{2})} - \frac{2\tan(\frac{\alpha}{2})}{1+\tan^2(\frac{\alpha}{2})}}{\frac{2\tan(\frac{\alpha}{2})}{1+\tan^2(\frac{\alpha}{2})} - \frac{1-\tan^2(\frac{\alpha}{2})}{1+\tan^2(\frac{\alpha}{2})}} = \dfrac{\frac{2\times\frac{1}{4}}{1-\frac{1}{16}} - \frac{2\times\frac{1}{4}}{1+\frac{1}{16}}}{\frac{2\times\frac{1}{4}}{1+\frac{1}{16}} - \frac{1-\frac{1}{16}}{1+\frac{1}{16}}}$ (9)

$$= \dfrac{\frac{\frac{1}{2}}{\frac{15}{16}} - \frac{\frac{1}{2}}{\frac{17}{16}}}{\frac{\frac{1}{2}}{\frac{17}{16}} - \frac{\frac{15}{16}}{\frac{17}{16}}} = \dfrac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}} = \dfrac{\frac{136-120}{255}}{\frac{-7}{17}} = -\frac{16\times17}{255\times7} = \boxed{\frac{-16}{105}}$$ ← جواب

$2\sin\alpha < \sin2\alpha \to 2\sin\alpha < 2\sin\alpha\cos\alpha \to 2\sin\alpha - 2\sin\alpha\cos\alpha < 0 \to 2\sin\alpha\underbrace{(1-\cos\alpha)}_{\text{نامنفی}} < 0$

$\frac{\cot\alpha}{\sin\alpha} > 0 \to \frac{\cos\alpha}{\sin^2\alpha} > 0 \to \cos\alpha > 0$ (1) 10

(2) $\sin\alpha < 0$

(1),(2) $\Rightarrow$ $\boxed{\text{در ربع چهارم است}}$ ← جواب