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نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس ۱۱A ۲۷

۱

$$S = \frac{1}{2} \times 4 \times \sqrt{3} \times \sin\alpha = 6/\Delta \Rightarrow \sin\alpha = \frac{9}{4\sqrt{3}} = \frac{\sqrt{3}}{2} \rightarrow \alpha = \begin{cases} \alpha = \frac{\pi}{3} \\ \alpha = \frac{2\pi}{3} \end{cases} \Rightarrow \frac{\alpha_{max}}{\alpha_{min}} = \frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = 2$$

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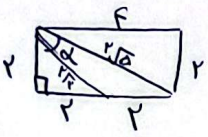

$$\frac{1}{2} \times \sin\alpha \times 2\sqrt{5} \times 2\sqrt{2} = \frac{1}{2} \times 2 \times 2 \Rightarrow \sin\alpha = \frac{\sqrt{10}}{10} \Rightarrow \cos\alpha = \frac{3\sqrt{10}}{10}$$

$$\Rightarrow \cot\alpha = \frac{3\sqrt{10}}{\sqrt{10}} = 3$$

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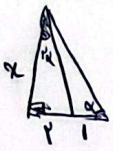

$$\tan 2\alpha = \frac{2}{x} \qquad \tan\alpha = \frac{x}{3} \Rightarrow \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \Rightarrow \frac{2}{x} = \frac{2\left(\frac{2x}{3}\right)}{1-\frac{x^2}{9}}$$

$$\Rightarrow \frac{2}{x} = \frac{\frac{4x}{3}}{\frac{9-x^2}{9}} \Rightarrow \frac{2}{x} = \frac{4x}{9-x^2} \Rightarrow \frac{1}{x} = \frac{2x}{9-x^2} \Rightarrow 9-x^2 = 2x^2 \Rightarrow 3x^2 = 9 \Rightarrow x = \frac{3}{\sqrt{3}}$$

$$\cot\alpha = \frac{3}{x} = \frac{3}{\frac{3}{\sqrt{3}}} = \sqrt{3}$$

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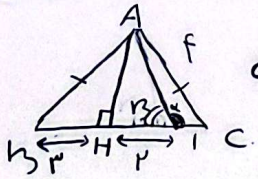
ABC متساوی‌الساقین $\Rightarrow AB = AC$



طبق فیثاغورس: $AH = \sqrt{AC^2 - CH^2} = \sqrt{4^2 - 3^2} = \sqrt{7}$

$$\tan\alpha = \tan(180 - B) = -\tan B = -\frac{\sqrt{7}}{2}$$

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$$2\sin^2 x + \cos^2 x = \frac{5}{3} \Rightarrow \sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} = \frac{5}{3} \Rightarrow \sin^2 x = \frac{1}{3} \Rightarrow \sin x$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\sin^2 x}{1-\sin^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

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$$\Rightarrow \frac{\sin^4(\alpha)+4(1-\sin^2(\alpha))}{1+(1-\sin^2(\alpha))} - \frac{\cos^4(\alpha)+4(1-\cos^2(\alpha))}{1+(1-\cos^2(\alpha))}$$

$$\Rightarrow \frac{\sin^4(\alpha)-4\sin^2(\alpha)+4}{2-\sin^2(\alpha)} - \frac{\cos^4(\alpha)-4\cos^2(\alpha)+4}{2-\cos^2(\alpha)} = \frac{(2-\sin^2(\alpha))^2}{2-\sin^2(\alpha)} - \frac{(2-\cos^2(\alpha))^2}{2-\cos^2(\alpha)}$$

$$\Rightarrow \frac{(2-\sin^2(\alpha))^2}{2-\sin^2(\alpha)} - \frac{(2-\cos^2(\alpha))^2}{2-\cos^2(\alpha)} = (2-\sin^2(\alpha)) - (2-\cos^2(\alpha)) = \cos^2(\alpha)-\sin^2(\alpha) = \cos 2\alpha$$

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$$\sin\left(\frac{9\pi}{2}+\alpha\right) = \sin\left(4\pi+\frac{\pi}{2}+\alpha\right) = \sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$$

$$\cos\left(\frac{7\pi}{2}-\alpha\right) = \cos\left(2\pi+\frac{3\pi}{2}-\alpha\right) = \cos\left(\frac{3\pi}{2}-\alpha\right) = -\sin\alpha$$

$$\tan\left(\alpha-\frac{3\pi}{2}\right) = -\tan\left(\frac{3\pi}{2}-\alpha\right) = -\cot\alpha$$

$\Rightarrow$ فیثاغورس : $x=4$ $\Rightarrow \cos\alpha = -\frac{3}{5}$, $-\sin\alpha = \frac{4}{5}$

$$\Rightarrow (\cos\alpha)(-\sin\alpha)-(-\cot\alpha) = -\frac{3}{5}+\frac{4}{5}+\frac{3}{4} = \frac{-12}{20}+\frac{3}{4} = \frac{-48+60}{100} = 0.12$$

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$$x=\frac{\pi}{12} \Rightarrow 3\cos\frac{\pi}{3}+\sqrt{2}\sin\frac{\pi}{12}-\sqrt{2}\cos\frac{\pi}{12} = \frac{3}{2}+\sqrt{2}\left(\sin\frac{\pi}{12}-\cos\frac{\pi}{12}\right)$$

$$A = \sin\frac{\pi}{12}-\cos\frac{\pi}{12} \xrightarrow{A<0} A^2 = \left(\sin\frac{\pi}{12}-\cos\frac{\pi}{12}\right)^2 = \sin^2\frac{\pi}{12}+\cos^2\frac{\pi}{12}-2\sin\frac{\pi}{12}\cos\frac{\pi}{12} \Rightarrow A^2 = 1-\sin\frac{\pi}{6}$$

$$\Rightarrow A^2 = 1-\frac{1}{2} \Rightarrow A = -\frac{1}{\sqrt{2}}$$

$\sin\frac{\pi}{12}$ چون $\cos\frac{\pi}{12}$ ضلع است

$$\Rightarrow \frac{3}{2}+\sqrt{2}\left(\sin\frac{\pi}{12}-\cos\frac{\pi}{12}\right) = \frac{3}{2}+\sqrt{2}\left(\frac{-1}{\sqrt{2}}\right) = \frac{3}{2}-1 = \frac{1}{2}$$

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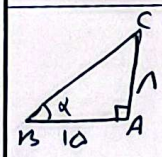

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{2\left(\frac{1}{4}\right)}{1-\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15}$$

فیثاغورس $= 17 = BC \rightarrow \begin{cases}\sin\alpha = \frac{8}{17} \\ \cos\alpha = \frac{15}{17}\end{cases} \Rightarrow \frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha} = \frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}} = \frac{-14}{105}$

۱۰

$$2\sin\alpha < \sin 2\alpha \Rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \Rightarrow 2\sin\alpha - 2\sin\alpha\cos\alpha < 0 \Rightarrow 2\sin\alpha(1-\cos\alpha) < 0$$

$\Rightarrow \sin\alpha < 0 \rightarrow$ $\sin\alpha$ زمانی منفی است که α در ربع ۳ و ۴ باشد

$\rightarrow \frac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow$ انتهای کمان α در ربع f است

$\cos\alpha < 0$ است