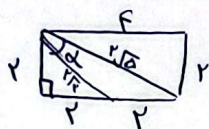


$$S = \frac{1}{2} \times 4 \times \sqrt{3} \times \sin \alpha = f, \alpha \Rightarrow \sin \alpha = \frac{9}{4\sqrt{3}} = \frac{\sqrt{3}}{4} \rightarrow \alpha = \begin{cases} \alpha = \frac{\pi}{4} \\ \alpha = \frac{3\pi}{4} \end{cases} \Rightarrow \frac{\alpha_{\max}}{\alpha_{\min}} = \frac{\frac{3\pi}{4}}{\frac{\pi}{4}} = 3$$



$$\frac{1}{2} \times \sin \alpha \times r\sqrt{2} \times r\sqrt{2} = \frac{1}{2} \times r^2 \times 2 \Rightarrow \sin \alpha = \frac{\sqrt{2}}{2} \Rightarrow \alpha = \frac{45^\circ}{1} \Rightarrow \alpha = \frac{45^\circ}{1}$$

$$\Rightarrow \alpha = \frac{45^\circ}{1} = 45^\circ$$

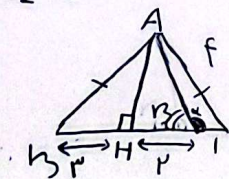


$$\tan 2\alpha = \frac{r}{x} \quad \tan \alpha = \frac{x}{r} \Rightarrow \tan 2\alpha = \frac{r \tan \alpha}{1 - \tan^2 \alpha} \Rightarrow \frac{r}{x} = \frac{r \left(\frac{x}{r}\right)}{1 - \frac{x^2}{r^2}}$$

$$\Rightarrow \frac{r}{x} = \frac{\frac{rx}{r}}{1 - \frac{x^2}{r^2}} \Rightarrow \frac{r}{x} = \frac{rx}{r - x^2} \Rightarrow \frac{1}{x} = \frac{rx}{r - x^2} \Rightarrow r - x^2 = rx^2 \Rightarrow rx^2 = r - x^2 \Rightarrow x^2 = \frac{r}{1+r} \Rightarrow x = \frac{r}{\sqrt{1+r}}$$

$$\Rightarrow \alpha = \frac{r}{x} = \frac{r}{\frac{r}{\sqrt{1+r}}} = \sqrt{1+r}$$

ABC $\Rightarrow AB = AC$



طبق قضیه فیثاغورس: $AH = \sqrt{AC^2 - CH^2} = \sqrt{f^2 - r^2} = \sqrt{5}$

$$\tan \alpha = \tan(180^\circ - B) = -\tan B = \frac{-\sqrt{5}}{r}$$

$$r \sin^2 x + \cos^2 x = \frac{f}{r} \Rightarrow \sin^2 x + \frac{\sin^2 x + \cos^2 x}{1} = \frac{f}{r} \Rightarrow \sin^2 x = \frac{1}{r} \Rightarrow \sin x$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\sin^2 x}{1 - \sin^2 x} = \frac{\frac{1}{r}}{1 - \frac{1}{r}} = \frac{1}{r-1}$$

$$\Rightarrow \frac{\sin^f(\alpha) + f(1 - \sin^f(\alpha))}{1 + (1 - \sin^f(\alpha))} - \frac{\cos^f(\alpha) + f(1 - \cos^f(\alpha))}{1 + (1 - \cos^f(\alpha))} = \frac{\sin^f(\alpha) - \cos^f(\alpha)}{1 - \sin^f(\alpha) - 1 + \cos^f(\alpha)}$$

$$\Rightarrow \frac{\sin^f(\alpha) - f\sin^f(\alpha) + f}{1 - \sin^f(\alpha)} - \frac{\cos^f(\alpha) - f\cos^f(\alpha) + f}{1 - \cos^f(\alpha)} = \frac{(1 - \sin^f(\alpha))^f}{1 - \sin^f(\alpha)} - \frac{(1 - \cos^f(\alpha))^f}{1 - \cos^f(\alpha)}$$

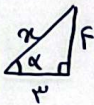
$$\Rightarrow \frac{(1 - \sin^f(\alpha))^f}{1 - \sin^f(\alpha)} - \frac{(1 - \cos^f(\alpha))^f}{1 - \cos^f(\alpha)} = (1 - \sin^f(\alpha)) - (1 - \cos^f(\alpha)) = \cos^f(\alpha) - \sin^f(\alpha) = \cos^f \alpha$$

f

$$\sin\left(\frac{9\pi}{4} + \alpha\right) = \sin\left(\pi + \frac{\pi}{4} + \alpha\right) = \sin\left(\frac{\pi}{4} + \alpha\right) = \cos \alpha$$

$$\cos\left(\frac{5\pi}{4} - \alpha\right) = \cos\left(\pi + \frac{\pi}{4} - \alpha\right) = \cos\left(\frac{\pi}{4} - \alpha\right) = -\sin \alpha$$

$$\tan\left(\alpha - \frac{\pi}{4}\right) = -\tan\left(\frac{\pi}{4} - \alpha\right) = -\cot \alpha$$



$$\Rightarrow \text{سید: } \alpha = \alpha \Rightarrow \cos \alpha = -\frac{a}{d}, \sin \alpha = \frac{f}{d}$$

$$\Rightarrow (\cos \alpha) (-\sin \alpha) - (-\cot \alpha) = -\frac{a}{d} + \frac{f}{a} + \frac{a}{f} = \frac{-af + a^2 + fd}{afd} = \frac{-f + d}{1} = \frac{d - f}{1} = \frac{f + d}{1}$$

v

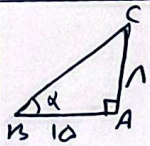
$$\frac{x = \frac{\pi}{4}}{\rightarrow} \sqrt{2} \cos \frac{\pi}{4} + \sqrt{2} \sin \frac{\pi}{4} - \sqrt{2} \cos \frac{\pi}{4} = \frac{\sqrt{2}}{\sqrt{2}} + \sqrt{2} \left(\sin \frac{\pi}{4} - \cos \frac{\pi}{4}\right)$$

$$A = \frac{\sin \frac{\pi}{4} - \cos \frac{\pi}{4}}{A} \Rightarrow A^2 = \left(\sin \frac{\pi}{4} - \cos \frac{\pi}{4}\right)^2 = \sin^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{4} - 2 \sin \frac{\pi}{4} \cos \frac{\pi}{4} \Rightarrow A^2 = 1 - \sin \frac{\pi}{2}$$

$$\Rightarrow A^2 = 1 - 1 \Rightarrow A = -\frac{1}{\sqrt{2}} \Rightarrow \frac{\sqrt{2}}{\sqrt{2}} + \sqrt{2} \left(\sin \frac{\pi}{4} - \cos \frac{\pi}{4}\right) = \frac{\sqrt{2}}{\sqrt{2}} + \sqrt{2} \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}}{\sqrt{2}} - 1 = \frac{1}{\sqrt{2}}$$

سید: $\sin \frac{\pi}{4} < \cos \frac{\pi}{4}$ منهای است

h



$$\tan \alpha = \frac{A \tan \frac{\alpha}{4}}{1 - \tan^2 \frac{\alpha}{4}} = \frac{A \left(\frac{1}{4}\right)}{1 - \frac{1}{16}} = \frac{\frac{A}{4}}{\frac{15}{16}} = \frac{A}{15}$$

$$\cos \frac{\alpha}{4} = \frac{14}{15} \Rightarrow \begin{cases} \sin \alpha = \frac{A}{15} \\ \cos \alpha = \frac{14}{15} \end{cases} \Rightarrow \frac{\tan \alpha - \sin \alpha}{\sin \alpha - \cos \alpha} = \frac{\frac{A}{15} - \frac{14}{15}}{\frac{14}{15} - \frac{A}{15}} = \frac{-14}{15 - A}$$

9

$$f \sin \alpha < \sin^f \alpha \Rightarrow f \sin \alpha < f \sin \alpha \cos \alpha \Rightarrow f \sin \alpha - f \sin \alpha \cos \alpha < 0 \Rightarrow f \sin \alpha (1 - \cos \alpha) < 0$$

$$\Rightarrow \sin \alpha < 0 \rightarrow f \sin \alpha < 0 \text{ و } 1 - \cos \alpha > 0 \text{ پس } f \sin \alpha < 0 \text{ است}$$

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