Year: Month: Day: Subject:

۱۸

۱) $\frac{1}{2} \times 6 \times \sqrt{3} \times \sin\alpha = 6\sqrt{3} \Rightarrow \sin\alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 120^\circ$ یا 60°

$\Rightarrow \max \alpha = 120^\circ$, $\min \alpha = 60^\circ \Rightarrow \frac{\max}{\min} = \frac{120}{60} = 2$ ✓

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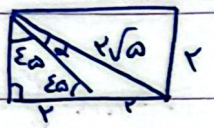
۲)



$\cos(45+\alpha) = \frac{2}{2\sqrt{5}}$ (I)

$\sin(45+\alpha) = \frac{4}{2\sqrt{5}}$ (II)

(I) $\Rightarrow \cos 45 \times \cos\alpha - \sin 45 \times \sin\alpha = \frac{\sqrt{5}}{5} \Rightarrow \cos\alpha - \sin\alpha = \frac{2\sqrt{5}}{5\sqrt{2}}$

(II) $\Rightarrow \sin 45 \times \cos\alpha - \cos 45 \times \sin\alpha = \frac{4\sqrt{5}}{5} \Rightarrow \sin\alpha + \cos\alpha = \frac{4\sqrt{5}}{5\sqrt{2}}$

$\Rightarrow \begin{cases} \cos\alpha - \sin\alpha = \frac{2\sqrt{5}}{5\sqrt{2}} \\ \sin\alpha + \cos\alpha = \frac{4\sqrt{5}}{5\sqrt{2}} \end{cases} \Rightarrow 2\cos\alpha = \frac{6\sqrt{5}}{5\sqrt{2}} \Rightarrow \cos\alpha = \frac{3\sqrt{5}}{5\sqrt{2}}$

$1 - \cos^2\alpha = \sin^2\alpha \Rightarrow 1 - \frac{45}{50} = \frac{5}{50} = \frac{1}{10} \Rightarrow \sin\alpha = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\cot\alpha = \frac{\frac{3\sqrt{5}}{5\sqrt{2}}}{\frac{\sqrt{10}}{10}} = \frac{3\sqrt{5} \times 10^2}{5\sqrt{2} \times \sqrt{10}} = \frac{6\sqrt{5}}{2\sqrt{5}} = 3$ ✓

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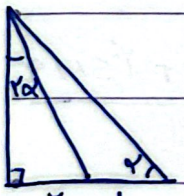
۳)



$\tan 2\alpha = \frac{2}{1}$ $\quad \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = 2$

$\Rightarrow 2\tan\alpha = 2 - 2\tan^2\alpha \Rightarrow 2\tan^2\alpha + 2\tan\alpha - 2$

$\xrightarrow{\tan\alpha = x} 2x^2 + 2x - 2 = 0 \Rightarrow \frac{-2 \pm \sqrt{20}}{4} = \begin{cases} \frac{\sqrt{5}-1}{2} \to \text{ق ق} \\ \frac{-\sqrt{5}-1}{2} \to \text{غ ق ق} \end{cases}$

$0 < \alpha < 90$

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۴) $\tan B = \frac{AD}{AB} \to \tan 2\alpha = \frac{2}{2x}$ $\quad \tan C = \frac{AB}{AC} \to \tan\alpha = \frac{2x}{3}$

$\tan 2\alpha \to \frac{2}{2x} = \frac{2 \times \frac{2x}{3}}{1 - \frac{4x^2}{9}} \to 2x = \frac{3}{2}$ $\quad \tan\alpha = \frac{1}{2}$, $\cot\alpha = 2$

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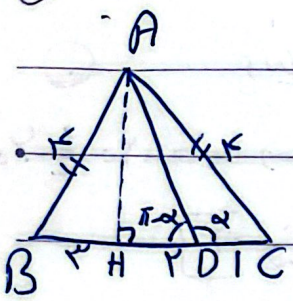

$AB^2 = AH^2 + BH^2 \Rightarrow 16 = AH^2 + 9 \Rightarrow AH^2 = 7$ (۴)

$\Rightarrow AH = \sqrt{7}$

$\tan(\pi - \alpha) = \frac{\sqrt{7}}{3} \Rightarrow -\tan\alpha = \frac{\sqrt{7}}{3}$ ؛ AHD مثلث

$\Rightarrow \tan\alpha = -\frac{\sqrt{7}}{3}$

$\sin^2 u + \sin^2 u + \underbrace{\cos^2 u}_{1} = \frac{4}{3} \Rightarrow \sin^2 u = \frac{1}{3} \Rightarrow \sin u = \frac{\sqrt{3}}{3}$ (۵)

$1 - \sin^2 u = \cos^2 u \Rightarrow 1 - \frac{1}{3} = \frac{2}{3} \Rightarrow \cos u = \frac{\sqrt{2}}{\sqrt{3}}$

$\tan^2 u = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} =$ (۶)

$\frac{(\sin^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{(\cos^2\alpha)^2 + 4\sin^2\alpha}{1+\sin^2\alpha} =$

$\frac{(1-\cos^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{(1-\sin^2\alpha) + 4\sin\alpha}{1+\sin^2\alpha} =$

$\frac{1 - 2\cos^2\alpha + \cos^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{1 - 2\sin^2\alpha + \sin^4\alpha + 4\sin\alpha}{1+\sin^2\alpha}$

$\Rightarrow \frac{1 + 2\cos^2\alpha + \cos^4\alpha}{1+\cos^2\alpha} - \frac{1 + 2\sin^2\alpha + \sin^4\alpha}{1+\sin^2\alpha} =$

$\frac{\cos^4\alpha + \cos^2\alpha}{1+\cos^2\alpha} - \frac{\sin^4\alpha + \sin^2\alpha}{1+\sin^2\alpha} = \frac{\cos^2\alpha(\cos^2\alpha + 1)}{1+\cos^2\alpha} - \frac{\sin^2\alpha(1+\sin^2\alpha)}{1+\sin^2\alpha}$

$= \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$

$\tan\alpha = \frac{4}{3}$ $\quad \alpha \longrightarrow$ ربع ۳ (۷)

$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right) = ?$ ۱/۷۵

$(\cos\alpha)(-\sin\alpha) - (-\cot\alpha) \longrightarrow (I)$

$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \Rightarrow 1+\frac{16}{9} = \frac{25}{9} \Rightarrow \cos\alpha = -\frac{3}{5}$

$1-\cos^2\alpha = \sin^2\alpha \Rightarrow 1-\frac{9}{25} = \frac{16}{25} = \frac{4}{5} = \sin\alpha$ ۲/۷۵

$(I) \Rightarrow \frac{-3}{5} \times \left(\frac{4}{5}\right) + \frac{3}{4} = \frac{-12}{5} + \frac{3}{4} = \frac{-48+15}{20} = \frac{-33}{20}$

$3\cos\frac{\pi}{3} + \sqrt{2}\sin\frac{\pi}{12} - \sqrt{2}\cos\frac{\pi}{12} = 3\left(\frac{1}{2}\right) + \sqrt{2}\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$ (۸)

$- \sqrt{2}\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) = \frac{3}{2} + \frac{\sqrt{12}}{4} - \frac{1}{2} - \frac{\sqrt{12}}{4} - \frac{1}{2} = \frac{1}{2}$

$\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\cos\frac{\pi}{4}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$ (۲)

$\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6} =$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

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(۹)

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \Rightarrow \tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} \Rightarrow \tan\alpha = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15}$

$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \Rightarrow 1+\frac{64}{225} = \frac{1}{\cos^2\alpha} = \frac{289}{225} = \frac{1}{\cos^2\alpha} \Rightarrow \cos\alpha = \frac{15}{17}$

$1-\cos^2\alpha = \frac{289}{289} - \frac{225}{289} = \frac{64}{289} = \frac{8}{17} = \sin\alpha$ ۱/۷۵

$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{16}{100}$

(۱۵) α در رابطه زیر صدق می‌کند

$2\sin\alpha < \sin^2\alpha$

$0 < \frac{\cot\alpha}{\sin\alpha} \Rightarrow \frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} \Rightarrow 0 < \frac{\cos\alpha}{\sin^2\alpha} \Rightarrow \cos\alpha > 0$

$0 < \sin^2\alpha - 2\sin\alpha \Rightarrow 0 < x^2 - 2x \Rightarrow \frac{0}{+} \frac{2}{-} \frac{}{+} \Rightarrow$

$-1 \leq \sin\alpha \leq 1 \Rightarrow -1 \leq \sin\alpha < 0 \Rightarrow$