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ریاضی یازدهم تک‌نسخه ①

$\frac{1}{2} \times 6 \times \sqrt{3} \times \sin\alpha = 4/5 \Rightarrow \sin\alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 120$ یا 60

$\Rightarrow \max\alpha = 120, \min\alpha = 60 \Rightarrow \frac{\max}{\min} = \frac{120}{60} = 2$

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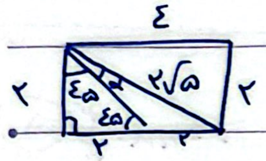
②



$\cos(45+\alpha) = \frac{2}{2\sqrt{5}}$ (I)

$\sin(45+\alpha) = \frac{4}{2\sqrt{5}}$ (II)

(I) $\Rightarrow \cos 45 \times \cos\alpha - \sin 45 \times \sin\alpha = \frac{\sqrt{5}}{5} \Rightarrow \cos\alpha - \sin\alpha = \frac{2\sqrt{5}}{5\sqrt{2}}$ ($\cos 45 = \frac{\sqrt{2}}{2}$, $\sin 45 = \frac{\sqrt{2}}{2}$)

(II) $\Rightarrow \sin 45 \times \cos\alpha - \cos 45 \times \sin\alpha = \frac{2\sqrt{5}}{5} \Rightarrow \sin\alpha + \cos\alpha = \frac{4\sqrt{5}}{5\sqrt{2}}$ ($\frac{\sqrt{2}}{2}$, $\frac{\sqrt{2}}{2}$)

$\Rightarrow \begin{cases} \cos\alpha - \sin\alpha = \frac{2\sqrt{5}}{5\sqrt{2}} \\ \sin\alpha + \cos\alpha = \frac{4\sqrt{5}}{5\sqrt{2}} \end{cases} \Rightarrow 2\cos\alpha = \frac{6\sqrt{5}}{5\sqrt{2}} \Rightarrow \cos\alpha = \frac{3\sqrt{5}}{5\sqrt{2}}$

$1 - \cos^2\alpha = \sin^2\alpha \Rightarrow 1 - \frac{45}{50} = \frac{5}{50} = \frac{1}{10} \Rightarrow \sin\alpha = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\cot\alpha = \frac{\frac{3\sqrt{5}}{5\sqrt{2}}}{\frac{\sqrt{10}}{10}} = \frac{3\sqrt{5} \times 10^{2}}{5 \times \sqrt{2} \times \sqrt{10}} = \frac{6\sqrt{5}}{2\sqrt{5}} = 3$

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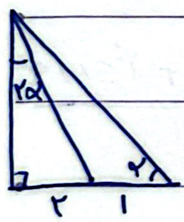
③



$\tan 2\alpha = \frac{2}{1} \quad \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = 2$

$\Rightarrow 2\tan\alpha = 2 - 2\tan^2\alpha \Rightarrow 2\tan^2\alpha + 2\tan\alpha - 2$

$\xRightarrow{\tan\alpha = x} 2x^2 + 2x - 2 = 0 \Rightarrow \frac{-2 \pm \sqrt{20}}{4} = \begin{cases} \frac{\sqrt{5}-1}{2} \to \text{ق ق} \\ \frac{-\sqrt{5}-1}{2} \to \text{غ ق ق} \end{cases}$

$0 < \alpha < 90$

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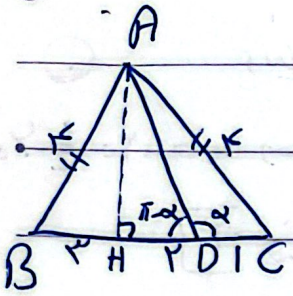

$AB^2 = AH^2 + BH^2 \Rightarrow 16 = AH^2 + 9 \Rightarrow AH^2 = 7$ (۴)

$\Rightarrow AH = \sqrt{7}$

$\tan(\pi - \alpha) = \frac{\sqrt{7}}{3} \Rightarrow -\tan\alpha = \frac{\sqrt{7}}{3}$ ∴ AHD مثلث

$\Rightarrow \tan\alpha = -\frac{\sqrt{7}}{3}$

$\sin^2 u + \sin^2 u + \underbrace{\cos^2 u}_{1} = \frac{4}{3} \Rightarrow \sin^2 u = \frac{1}{3} \Rightarrow \sin u = \frac{\sqrt{3}}{3}$ (۵)

$1 - \sin^2 u = \cos^2 u \Rightarrow 1 - \frac{1}{3} = \frac{2}{3} \Rightarrow \cos u = \frac{\sqrt{2}}{\sqrt{3}}$

$\tan^2 u = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} =$ (۶)

$\frac{(\sin^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{(\cos^2\alpha)^2 + 4\sin^2\alpha}{1+\sin^2\alpha} =$

$\frac{(1-\cos^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{(1-\sin^2\alpha) + 4\sin\alpha}{1+\sin^2\alpha} =$

$\frac{1 - 2\cos^2\alpha + \cos^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{1 - 2\sin^2\alpha + \sin^4\alpha + 4\sin\alpha}{1+\sin^2\alpha}$

$\Rightarrow \frac{1 + 2\cos^2\alpha + \cos^4\alpha}{1+\cos^2\alpha} - \frac{1 + 2\sin^2\alpha + \sin^4\alpha}{1+\sin^2\alpha} =$

$\frac{\cos^4\alpha + \cos^2\alpha}{1+\cos^2\alpha} - \frac{\sin^4\alpha + \sin^2\alpha}{1+\sin^2\alpha} = \frac{\cos^2\alpha(\cos^2\alpha + 1)}{1+\cos^2\alpha} - \frac{\sin^2\alpha(1+\sin^2\alpha)}{1+\sin^2\alpha}$

$= \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$

$\tan\alpha = \frac{4}{3}$ $\alpha \longrightarrow$ ربع ۳ (۷)

$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right)-\tan\left(\alpha-\frac{3\pi}{2}\right)=?$

$(\cos\alpha)(-\sin\alpha)-(-\cot\alpha) \longrightarrow (I)$

$1+\tan^2\alpha=\frac{1}{\cos^2\alpha} \Rightarrow 1+\frac{16}{9}=\frac{25}{9} \Rightarrow \cos\alpha=-\frac{3}{5}$

$1-\cos^2\alpha=\sin^2\alpha \Rightarrow 1-\frac{9}{25}=\frac{16}{25}=\frac{4}{5}=\sin\alpha$

$(I) \Rightarrow \frac{-3}{5}\times\left(\frac{4}{5}\right)+\frac{3}{4}=\frac{-12}{5}+\frac{3}{4}=\frac{-48+15}{20}=\boxed{\frac{-33}{20}}$

$3\cos\frac{\pi}{3}+\sqrt{2}\sin\frac{\pi}{12}-\sqrt{2}\cos\frac{\pi}{12}=3\left(\frac{1}{2}\right)+\sqrt{2}\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$ (۸)

$-\sqrt{2}\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)=\frac{3}{2}+\frac{\sqrt{12}}{4}-\frac{1}{2}-\frac{\sqrt{12}}{4}-\frac{1}{2}=\boxed{\frac{1}{2}}$

$\sin\frac{\pi}{12}=\sin\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\sin\frac{\pi}{4}\cos\frac{\pi}{6}-\sin\frac{\pi}{6}\cos\frac{\pi}{4}$

$=\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}=$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

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(۹)

$\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha} \Rightarrow \tan\alpha=\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} \Rightarrow \tan\alpha=\frac{2\times\frac{1}{4}}{1-\frac{1}{16}}=\frac{\frac{1}{2}}{\frac{15}{16}}=\frac{8}{15}$

$1+\tan^2\alpha=\frac{1}{\cos^2\alpha} \Rightarrow 1+\frac{64}{225}=\frac{1}{\cos^2\alpha}=\frac{289}{225}=\frac{1}{\cos^2\alpha} \Rightarrow \cos\alpha=\frac{15}{17}$

$1-\cos^2\alpha=\frac{289}{289}-\frac{225}{289}=\frac{64}{289}=\frac{8}{17}=\sin\alpha$

$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=\boxed{\frac{16}{105}}$

(10) α در ربع چهارم است.

$2\sin\alpha < \sin^2\alpha$

$0 < \frac{\cot\alpha}{\sin\alpha} \Rightarrow \frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} \Rightarrow 0 < \frac{\cos\alpha}{\sin^2\alpha} \Rightarrow \cos\alpha > 0$

$0 < \sin^2\alpha - 2\sin\alpha \Rightarrow 0 < x^2 - 2x \Rightarrow \frac{0}{2} \quad \frac{2}{|-|+} \Rightarrow$

$-1 \leq \sin\alpha \leq 1 \Rightarrow -1 \leq \sin\alpha < 0 \Rightarrow$