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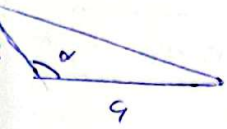
آلا کرمانی — پاسخنامه تشریحی ۲۷

①



$\frac{1}{2}\times 9\times\sqrt{3}\times \sin\alpha = 18\sqrt{3} \Rightarrow \sin\alpha = \frac{36\sqrt{3}}{18\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$ → $\sin 120^\circ$ min, $\sin 60^\circ$ min

$\frac{120}{60} = 2$

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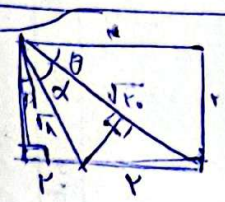
②



$\alpha = 90 - (\theta+\beta) \Rightarrow \cot\alpha = \cot(90-(\theta+\beta)) = \tan(\theta+\beta) =$

$\tan(\theta+\beta) = \frac{\tan\theta+\tan\beta}{1-\tan\theta\tan\beta} = \frac{\frac{1}{2}+1}{1-\frac{1}{2}} = \frac{\frac{3}{2}}{\frac{1}{2}} = \frac{3}{1} = 3$

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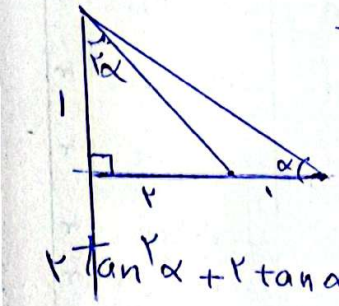
③



$\tan 2\alpha = \frac{2}{1}$

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = 2 \Rightarrow 2\tan\alpha = 2 - 2\tan^2\alpha \Rightarrow$

$2\tan^2\alpha + 2\tan\alpha - 2 \Rightarrow 2x^2 + 2x - 2 = 0 \Rightarrow \frac{-2\pm\sqrt{4+16}}{4} =$ ($\tan = x$) → $\frac{\sqrt{5}-1}{2}$ ق ق ، $\frac{-\sqrt{5}-1}{2}$ غ ق ق

$(0<\alpha<90)$

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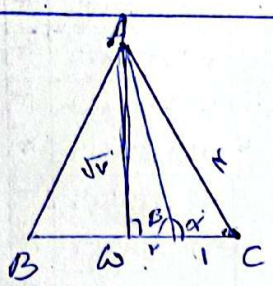
④



$\tan\alpha = \tan(180-\beta) = -\tan\beta \Rightarrow \tan\alpha = -\frac{\sqrt{7}}{2}$

$\tan\beta = \frac{\sqrt{7}}{2}$

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⑤

$\sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{\frac{4}{3}} = \frac{4}{3} \Rightarrow \sin^2 x = \frac{1}{3} \Rightarrow \sin x = \pm\frac{\sqrt{3}}{3}$ ، $2\sin^2 x + \cos^2 x = \frac{4}{3}$

$\cos^2 x + \frac{1}{3} = 1 \Rightarrow \cos^2 x = \frac{2}{3} \Rightarrow \cos x = \pm\sqrt{\frac{2}{3}} \Rightarrow$ $\tan^2\alpha = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

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۳) $\tan B = \frac{AD}{AB} \to \tan 2\alpha = \frac{2}{x}$ $\quad$ $\tan C = \frac{AB}{AC} \to \tan\alpha = \frac{x}{3}$

$\to \tan 2\alpha \to \frac{2}{x} = \frac{2\times\frac{x}{3}}{1-\frac{x^2}{9}} \to x = \frac{3}{2}$ $\quad$ $\tan\alpha = \frac{1}{2}$, $\cot\alpha = 2$

CamScanner

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$\tan\alpha = \frac{4}{3}\frac{\sin\alpha}{\cos\alpha}$ ، $\frac{\sin\alpha}{\cos\alpha} = \frac{4m}{3m}$ ، $16m^2 + 9m^2 = 1 \Rightarrow 25m^2 = 1 \Rightarrow m^2 = \frac{1}{25} \Rightarrow m = \pm\frac{1}{5}$ ، تانژانت α

$\sin = \frac{-4}{5}$ ، $\cos = \frac{-3}{5}$

$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha - \frac{13\pi}{2}\right) = \sin\left(\frac{\pi}{2}+\alpha\right)\cos\left(\frac{\pi}{2}-\alpha\right) - \tan(\alpha-$

$(+\cos\alpha - \sin\alpha) - (-\cot\alpha) = \left(+\frac{3}{5} + \frac{4}{5}\right) + \left(\frac{3}{4}\right) = \frac{12}{20} + \frac{3}{4} = \frac{27}{20} = 0/27$

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$3\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x \xrightarrow{x=\frac{\pi}{12}} 3\cos\frac{\pi}{3} + \sqrt{2}\sin\frac{\pi}{12} - \sqrt{2}\cos x =$

$\sin\frac{\pi}{12} = \sin(45-30) = \sin 45\cos 30 - \sin 30\cos 45 = \frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\times\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos\frac{\pi}{12} = \cos(45-30) = \cos 45\cos 30 + \sin 30\sin 45 = \frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\times\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\frac{3}{2} + \sqrt{2}(\sin x - \cos x) = \frac{3}{2} + \sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = \frac{1}{2}$

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$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \Rightarrow \tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} \Rightarrow \tan\alpha = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15}$

$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \Rightarrow 1 + \frac{64}{225} = \frac{1}{\cos^2\alpha} = \frac{289}{225} = \frac{1}{\cos^2\alpha} \Rightarrow \cos^2\alpha = \frac{225}{289} \Rightarrow \cos\alpha = \pm\frac{15}{17}$

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$1-\cos^2\alpha = \frac{289}{289} - \frac{225}{289} = \frac{64}{289} = \pm\frac{8}{17} = \sin\alpha$

$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\sin\alpha - \sin\alpha\cos\alpha}{\sin\cos\alpha - \cos^2\alpha} = \frac{-19}{150}$

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$2\sin\alpha < \sin 2\alpha$

$0 < \frac{\cot\alpha}{\sin\alpha} \Rightarrow \frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} = 0 < \frac{\cos\alpha}{\sin^2\alpha} \Rightarrow \cos\alpha > 0 \Rightarrow$ اول یا چهارم

$0 < \sin^2\alpha - 2\sin\alpha$ ، $\begin{array}{c|c|c} 0 & & 2 \\ + & - & + \end{array} \Rightarrow -1 < \sin\alpha < 1 \Rightarrow 0 < \sin\alpha < 1 \Rightarrow$

α در ربع چهارم

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$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{\sin^4\alpha + 4(1-\sin^2\alpha)}{2-\sin^2\alpha} - \frac{\cos^4\alpha + 4(1-\cos^2\alpha)}{2-\cos^2\alpha}$

$\frac{\sin^4\alpha + 4 - 4\sin^2\alpha}{2-\sin^2\alpha} - \frac{\cos^4\alpha + 4 - 4\cos^2\alpha}{2-\cos^2\alpha} = \frac{(\sin^2\alpha - 2)^2}{-(\sin^2\alpha - 2)} - \frac{(\cos^2\alpha - 2)^2}{-(\cos^2\alpha - 2)} = (\sin^2\alpha - 2) - (\cos^2\alpha - 2)$

$\cos^2\alpha - \sin^2\alpha = \cos 2\alpha$