آرالا کریمی پاسخنامه تست ریاضی ۲۷

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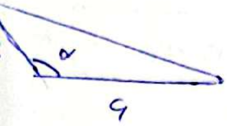

$$\frac{1}{2}\times 6\times\sqrt{3}\times\sin\alpha = 13.5 \Rightarrow \sin\alpha = \frac{45}{30\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$$

$\sin 120^\circ$ max

$\sin 60^\circ$ min

$$\frac{120}{60} = 2$$

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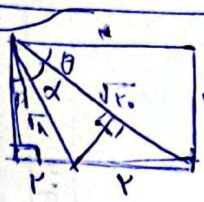

$$\alpha = 90 - (\theta+\beta) \rightarrow \cot\alpha = \cot(90-(\theta+\beta)) = \tan(\theta+\beta) =$$

$$\tan(\theta+\beta) = \frac{\tan\theta+\tan\beta}{1-\tan\theta\tan\beta} = \frac{\frac{1}{2}+1}{1-\frac{1}{2}} = \frac{\frac{3}{2}}{\frac{1}{2}} = \frac{3}{1} = 3$$

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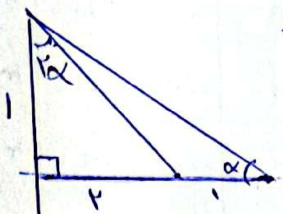

$$\tan 2\alpha = \frac{2}{1}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = 2 \Rightarrow 2\tan\alpha = 2-2\tan^2\alpha \Rightarrow$$

$$2\tan^2\alpha + 2\tan\alpha - 2 \underset{\tan = x}{\Rightarrow} 2x^2+2x-2 = 0 \Rightarrow \frac{-2\pm\sqrt{4+16}}{4} =$$

$\frac{\sqrt{5}-1}{2}$ ق ق

$\frac{-\sqrt{5}-1}{2}$ غ ق ق

$(0<\alpha<90)$

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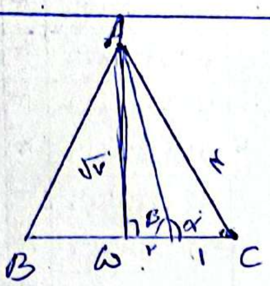

$$\tan\alpha = \tan(180-\beta) = -\tan\beta \Rightarrow \tan\alpha = -\frac{\sqrt{7}}{2}$$

$$\tan\beta = \frac{\sqrt{7}}{2}$$

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$$2\sin^2 x + \cos^2 x = \frac{4}{3}$$

$$\sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} = \frac{4}{3} \Rightarrow \sin^2 x = \frac{1}{3} \Rightarrow \sin x = \pm\frac{\sqrt{3}}{3}$$

$$\cos^2 x + \frac{1}{3} = 1 \Rightarrow \cos^2 x = \frac{2}{3} \Rightarrow \cos x = \pm\sqrt{\frac{2}{3}} \Rightarrow$$

$$\tan^2\alpha = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

(7)

$\tan\alpha = \frac{4}{3} = \frac{\sin\alpha}{\cos\alpha}$, [illegible] $\left| \frac{\sin\alpha}{\cos\alpha} = \frac{4x}{3x} \Rightarrow 16x^2 + 9x^2 = 1 \Rightarrow 25x^2 = 1 \Rightarrow x^2 = \frac{1}{25} \Rightarrow x = \pm\frac{1}{5}\right.$

$\sin = \frac{-4}{5}$, $\cos = \frac{-3}{5}$

$\sin\left(\frac{9\pi}{2} + \alpha\right)\cos\left(\frac{7\pi}{2} - \alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right) = \sin\left(\frac{\pi}{2} + \alpha\right)\cos\left(\frac{3\pi}{2} - \alpha\right) - \tan(\alpha -$

$(+\cos\alpha \cdot -\sin\alpha) - (\cot\alpha) = \left(+\frac{3}{5} \times +\frac{4}{5}\right) - \left(\frac{3}{4}\right) = \frac{12}{25} + \frac{3}{4} = \frac{27}{100} = 0{,}27$

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(8)

$3\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x \xrightarrow{x = \frac{\pi}{12}} 3\cos\frac{\pi}{3} + \sqrt{2}\sin\frac{\pi}{12} - \sqrt{2}\cos x =$

$\sin\frac{\pi}{12} = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \sin 30^\circ \cos 45^\circ = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$

$\cos\frac{\pi}{12} = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 30^\circ \sin 45^\circ = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$

$\frac{3}{2} + \sqrt{2}(\sin x - \cos x) = \frac{3}{2} + \sqrt{2}\left(-\frac{\sqrt{2}}{4}\right) = \frac{1}{2}$

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(9)

$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha} \Rightarrow \tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1 - \tan^2\frac{\alpha}{2}} \Rightarrow \tan\alpha = \frac{2 \times \frac{1}{4}}{1 - \frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15}$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \Rightarrow 1 + \frac{64}{225} = \frac{1}{\cos^2\alpha} = \frac{289}{225} = \frac{1}{\cos^2\alpha} \Rightarrow \cos^2\alpha = \frac{225}{289} \Rightarrow \cos\alpha = \pm\frac{15}{17}$

$1 - \cos^2\alpha = \frac{289}{289} - \frac{225}{289} = \frac{64}{289} = \pm\frac{8}{17} = \sin\alpha$

$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\sin\alpha - \sin\alpha\cos\alpha}{\sin\alpha\cos\alpha - \cos^2\alpha} = \frac{69}{105}$

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(10)

$2\sin\alpha < \sin^2\alpha$

$0 < \frac{\cot\alpha}{\sin\alpha} \Rightarrow \frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} = 0 < \frac{\cos\alpha}{\sin^2\alpha} \Rightarrow \cos\alpha > 0 \Rightarrow$ اول یا چهارم

$0 < \sin^2\alpha - 2\sin\alpha \Rightarrow 0 < x^2 - 2x$: $\begin{array}{c|c|c} & 0 \quad\quad 2 & \\ + & - & + \end{array} \Rightarrow -1 \leq \sin\alpha \leq 1 \Rightarrow -1 \leq \sin\alpha < 0 \Rightarrow$ 

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(6)

$\frac{\sin^4\alpha + 4\cos^2\alpha}{1 + \cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1 + \sin^2\alpha} = \frac{\sin^4\alpha + 4(1 - \sin^2\alpha)}{2 - \sin^2\alpha} - \frac{\cos^4\alpha + 4(1 - \cos^2\alpha)}{2 - \cos^2\alpha}$

$\frac{\sin^4\alpha + 4 - 4\sin^2\alpha}{2 - \sin^2\alpha} - \frac{\cos^4\alpha + 4 - 4\cos^2\alpha}{2 - \cos^2\alpha} = \frac{(\sin^2\alpha - 2)^2}{-(\sin^2\alpha - 2)} - \frac{(\cos^2\alpha - 2)^2}{-(\cos^2\alpha - 2)} = -(\sin^2\alpha - 2) + (\cos^2\alpha - 2)$

$\cos^2\alpha - \sin^2\alpha = \cos 2\alpha$